Mapping surface displacement using a pair of interferograms: a comparative study

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Introduction

- InSAR analysis → 1D measurement: projection of the surface deformation field into the Line-of-sight (LOS)

- To resolve the 3D deformation field with standard InSAR techniques, a combination of at least 3 interferograms acquired with different imaging geometries is required.

- However, most areas are regularly imaged by two configurations: one ascending, one descending.

Gonzalez et al 2009, Bam earthquake, Iran
Introduction

Equation system defined for a pair of interferograms

\[
\begin{pmatrix}
d_{ASC} \\
d_{DESC}
\end{pmatrix} =
\begin{pmatrix}
-sin(\theta_A) \sin(\tilde{\alpha}_A) & -sin(\theta_A) \cos(\tilde{\alpha}_A) & \cos(\theta_A) \\
-sin(\theta_D) \sin(\tilde{\alpha}_D) & -sin(\theta_D) \cos(\tilde{\alpha}_D) & \cos(\theta_D)
\end{pmatrix}
\begin{pmatrix}
U_e \\
U_n \\
U_{up}
\end{pmatrix}
\]

\(\theta\) is the incidence angle, \(\tilde{\alpha}_A\) and \(\tilde{\alpha}_D\) are the azimuth look directions

A linear system of equations

\[
d = Gm
\]

Observations  
Model
Decomposition approaches

• 2 component linear inversion:

Hypothesis on the nature of the deformation field: Elimination of 1 or 2 horizontal components

\[
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  U_e \\
  U_{up}
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\]

\[U_{up} = \frac{d_{LOS}}{\cos(\theta)}\]

• Linear combination (LC method):

Linear combination applied on the LOS unit vectors → sensitivity to near-vertical and near-east components

Near-Up: \[NU = d_{DESC} + d_{ASC}\]

Near-East: \[NE = d_{DESC} - d_{ASC}\]
Objectives

1- Quantify the **ability to reconstruct the components of the true deformation field** using a pair of interferograms and the **model resolution matrix**

2- **Propose a robust method** that takes into account uncertainties of the true deformation field measurement to reconstruct the vertical and east-components using a pair of interferograms

3- **Compare** our approach with the classical decomposition methods
Model Resolution Matrix (MRM)

- System of acquisition (side-looking geometry) does not measure the true deformation field → it acts as a spatial filter

- Is it possible to estimate the true deformation field using the information on acquisition system (G matrix) Or is it possible to estimate an error on the component retrieval?

\[
\begin{pmatrix}
d_{ASC} \\
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-sin(\theta_D)sin(\tilde{\alpha}_D) & -sin(\theta_D)cos(\tilde{\alpha}_D) & cos(\theta_D)
\end{pmatrix}
\begin{pmatrix}
U_e \\
U_n \\
U_{wp}
\end{pmatrix}
\]

\[m^{est} = \left[G^{-g}G\right]m^{true} = Rm^{true}\]

Inverse general matrix
Model Resolution Matrix: Application

Bárðarbunga (Iceland) CosmoSky-Med data

True deformation field

<table>
<thead>
<tr>
<th></th>
<th>$U_{up}$</th>
<th>$U_n$</th>
<th>$U_{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{up}^*$</td>
<td>0.9283</td>
<td>-0.2576</td>
<td>0.0139</td>
</tr>
<tr>
<td>$U_n^*$</td>
<td>-0.2576</td>
<td>0.0744</td>
<td>0.0498</td>
</tr>
<tr>
<td>$U_{e}^*$</td>
<td>0.0139</td>
<td>0.0498</td>
<td>0.9973</td>
</tr>
</tbody>
</table>

$U_{up}^* = 0.9283 U_{up} - 0.2576 U_n + 0.0139 U_e$

$U_n^* = -0.2576 U_{up} + 0.0744 U_n + 0.0498 U_e$

$U_{e}^* = 0.0139 U_{up} + 0.0498 U_n + 0.9973 U_e$
Inversion

General solution of the linear inverse problem:

\[
m^{est} = G^{-g} d^{obs}
\]

G is decomposed using a SVD.

To construct the general inverse we truncate the initial decomposition by taking into account only the eigenvalues containing information.

The general inverse can be written as follow (Menke, 1989):

\[
m^{est} = \underbrace{V_p \Lambda_p^{-1} U_p^T}_G d^{obs}
\]
Bárðarbunga

Interferograms
Retrieved Vertical component
Retrieved East component

Vertical component
East component

LC method
2 comp. linear inversion
SVD-based method
Comparison using Simulations

Mogi source: \( dV = 0.1 \text{ km}^3 \)
\( Z = 4 \text{ km} \)
Detailed comparison: $U_z$

- Incidence angles different by 20° as TerraSAR-X data
- Oblique incidence angles different by 10° as CosmoSky-Med data

Retrieved Vertical component

- Dip-Slip EW
- Strike-Slip EW
- Dike EW
- Strike-Slip NS
- Dip-Slip NS

Decrease in the Vertical component / Increase in the horizontal components
Detailed comparison: $U_E$

- Similar incidence angles as Keiding et al. (2010)
- Incidence angles different by 20° as TerraSAR-X data
- Oblique incidence angles different by 10° as CosmoSky-Med data

Retrieved East component

- Dip-Slip EW
- Dike EW
- Strike-Slip EW

- Dip-Slip NS
- Dike NS
- Strike-Slip NS

Decrease in the Vertical component / Increase in the horizontal components

Normalized cumulative error

Decomposition method

- Linear Combination
- Two comp. linear inversion
- SVD based inversion

$\frac{U_z}{U_h} = 1.3$ Mogi

$\frac{U_z}{U_h} = 0.9$

$\frac{U_z}{U_h} = 0.5$

$\frac{U_z}{U_h} = 0.1$
Comparison using simple Modelling

- Modelling based on Decomposition methods

Mogi source: $dV = 0.1 \text{ km}^3$
$Z = 4 \text{ km}$
Comparison using simple Modelling

**Modelling based on Decomposition methods**

- **Mogi source:** 
  - $dV = 0.1 \text{ km}^3$
  - $Z = 4 \text{ km}$
Summary

- Decomposition results will depend on: the combined viewing geometries, the deformation field and the orientation of its source.
- Mixed incidence angles, contributes to reduce errors on the reconstructed east and vertical components.
- LC method: not particularly recommended.
- Model resolution matrix quantifies the uncertainties on the true deformation field measurement and can be used for better constraining models.