Mapping surface displacement using a pair of interferograms: a comparative study

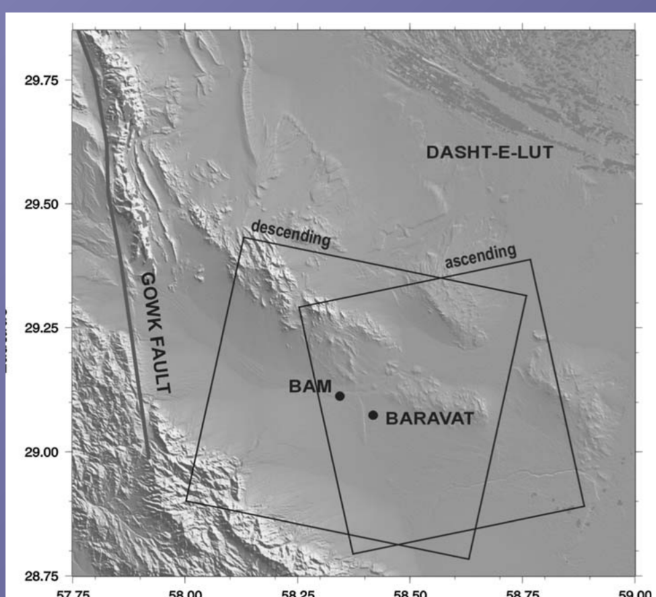
S. Dumont[1,2], F. Sigmundsson[1], F. Lopes[3]

1 Institute of Earth Sciences, University of Iceland, Reykjavík, Iceland
2 IDL, University of Beira Interior, Covilhã, Portugal
3 Institut de Physique du Globe, Paris, France

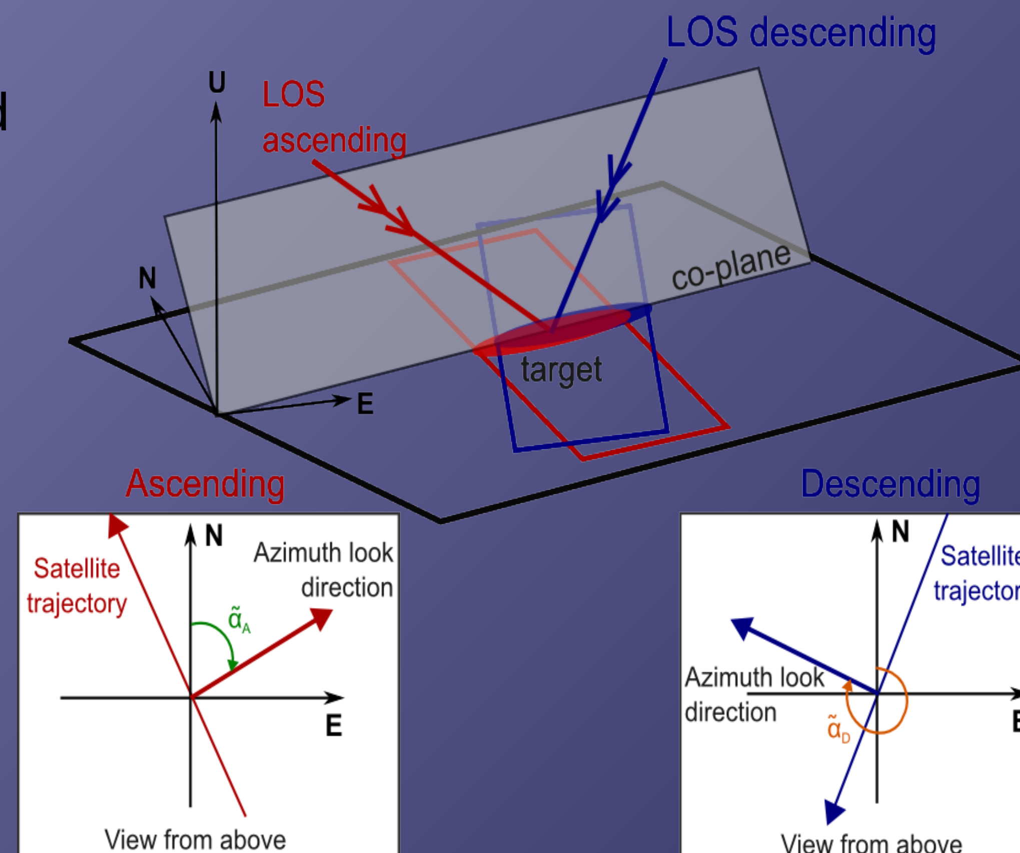
FUTUREVOLC
A EUROPEAN VOLCANOLOGICAL SUPERSITE IN ICELAND:
A MONITORING SYSTEM AND NETWORK FOR THE FUTURE

Introduction

- InSAR analysis → 1D measurement: projection of the surface deformation field into the Line-of-sight (LOS)
- To resolve the 3D deformation field with standard InSAR techniques, a combination of at least 3 interferograms acquired with different imaging geometries is required
- However, most areas are regularly imaged by two configurations: one ascending, one descending



Gonzalez et al 2009, Bam earthquake, Iran



Introduction

Equation system defined for a pair of interferograms

$$\begin{pmatrix} d_{ASC} \\ d_{DESC} \end{pmatrix} = \begin{pmatrix} -\sin(\theta_A)\sin(\tilde{\alpha}_A) & -\sin(\theta_A)\cos(\tilde{\alpha}_A) & \cos(\theta_A) \\ -\sin(\theta_D)\sin(\tilde{\alpha}_D) & -\sin(\theta_D)\cos(\tilde{\alpha}_D) & \cos(\theta_D) \end{pmatrix} \begin{pmatrix} U_e \\ U_n \\ U_{up} \end{pmatrix}$$

θ is the incidence angle, $\tilde{\alpha}_A$ and $\tilde{\alpha}_D$ are the azimuth look directions

A linear system of equations

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

Model

Observations

Decomposition approaches

- 2 component linear inversion:

Hypothesis on the nature of the deformation field: Elimination of 1 or 2 horizontal components

$$\begin{pmatrix} d_{ASC} \\ d_{DESC} \end{pmatrix} = \begin{pmatrix} -\sin(\theta_A)\sin(\tilde{\alpha}_A) & -\sin(\theta_A)\cos(\tilde{\alpha}_A) & \cos(\theta_A) \\ -\sin(\theta_D)\sin(\tilde{\alpha}_D) & -\sin(\theta_D)\cos(\tilde{\alpha}_D) & \cos(\theta_D) \end{pmatrix} \begin{pmatrix} U_e \\ U_n \\ U_{up} \end{pmatrix} \Rightarrow \begin{cases} \begin{pmatrix} d_{ASC} \\ d_{DESC} \end{pmatrix} = \begin{pmatrix} -\sin(\theta_A)\sin(\tilde{\alpha}_A) & \cos(\theta_A) \\ -\sin(\theta_D)\sin(\tilde{\alpha}_D) & \cos(\theta_D) \end{pmatrix} \begin{pmatrix} U_e \\ U_{up} \end{pmatrix} \\ U_{up} = d_{LOS}/cos\theta \end{cases}$$

- Linear combination (LC method):

Linear combination applied on the LOS unit vectors → sensitivity to near-vertical and near-east components

$$\text{Near-Up:} \quad \mathrm{NU} = \mathrm{d}_{DESC} + \mathrm{d}_{ASC}$$

$$\text{Near-East:} \quad \mathrm{NE} = \mathrm{d}_{DESC} - \mathrm{d}_{ASC}$$

Objectives

1- Quantify the **ability to reconstruct the components of the true deformation field** using a pair of interferograms and the **model resolution matrix**

2- **Propose a robust method** that takes into account uncertainties of the true deformation field measurement to reconstruct the vertical and east-components using a pair of interferograms

3- **Compare** our approach with the classical decomposition methods

Model Resolution Matrix (MRM)

- System of acquisition (side-looking geometry) does not measure the true deformation field → it acts as a spatial filter

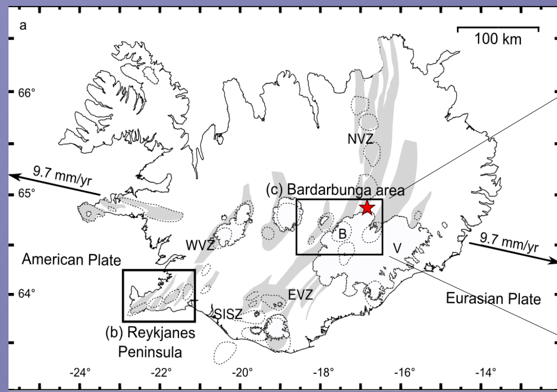
- Is it possible to estimate the true deformation field using the information on acquisition system (G matrix) Or is it possible to estimate an error on the component retrieval ?

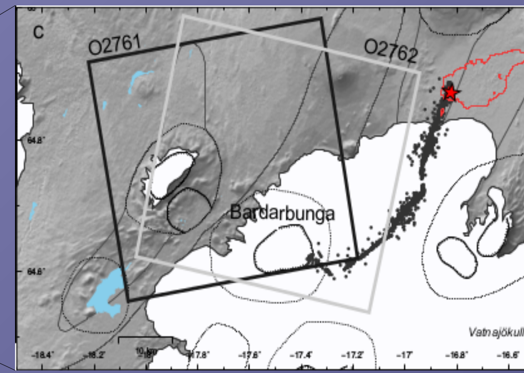
$$\begin{pmatrix} d_{ASC} \\ d_{DESC} \end{pmatrix} = \underbrace{\begin{pmatrix} -\sin(\theta_A)\sin(\tilde{\alpha}_A) & -\sin(\theta_A)\cos(\tilde{\alpha}_A) & \cos(\theta_A) \\ -\sin(\theta_D)\sin(\tilde{\alpha}_D) & -\sin(\theta_D)\cos(\tilde{\alpha}_D) & \cos(\theta_D) \end{pmatrix}}_{\mathbf{G}} \left.\begin{pmatrix} U_e \\ U_n \\ U_{up} \end{pmatrix}\right\} \mathbf{m}^{\mathbf{est}}$$

$$\mathbf{m}^{est} = [\mathbf{G}^{-g}\mathbf{G}]\mathbf{m}^{true} = \mathbf{R}\mathbf{m}^{true}$$

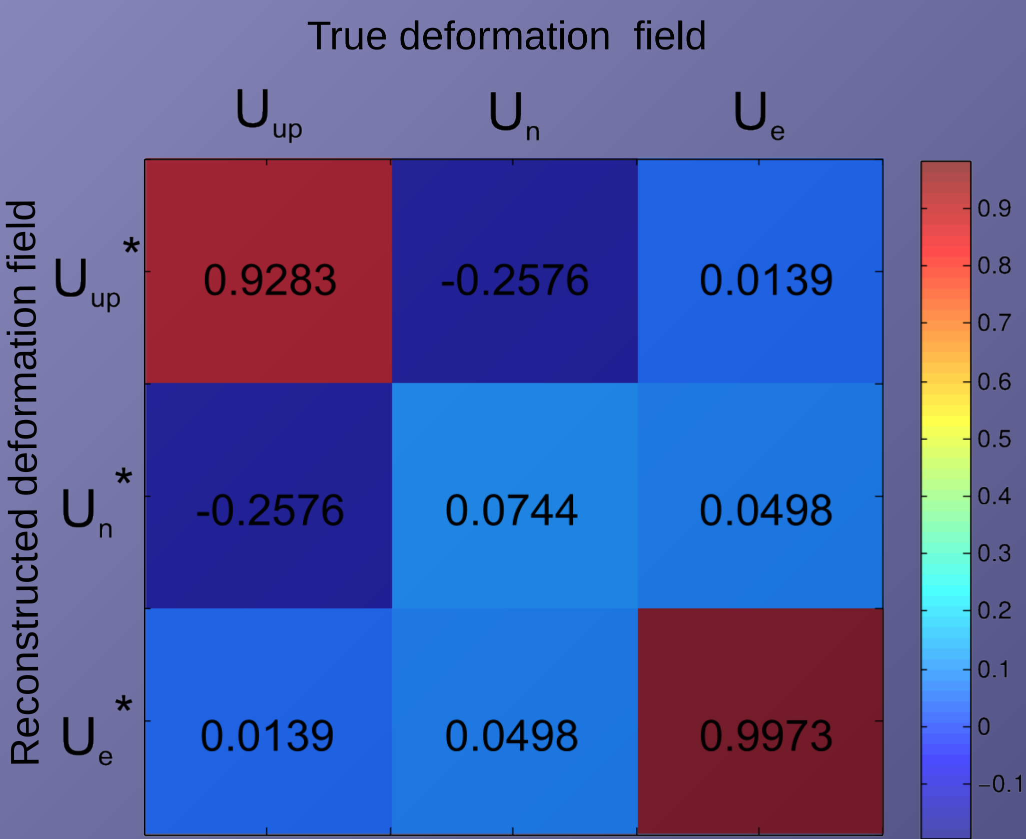
Inverse general matrix (→ $\mathbf{G}^{-g}$)

Model Resolution Matrix: Application





Bárðarbunga (Iceland)
CosmoSky-Med data



$$U_{UP}^* = 0.9283\, U_{UP} - 0.2576\, U_N + 0.0139\, U_E$$

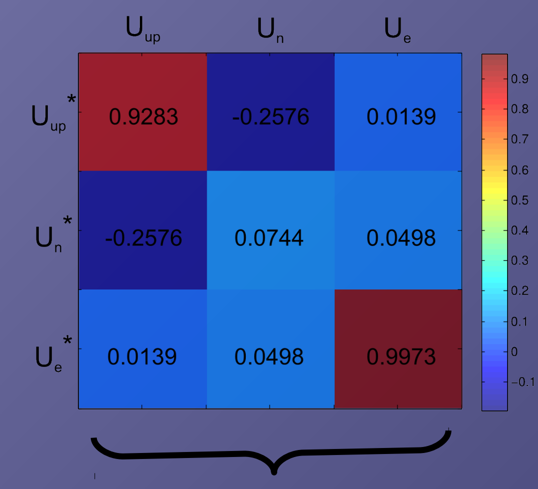
$$U_N^* = -0.2576\, U_{UP} + 0.0744\, U_N + 0.0498\, U_E$$

$$U_E^* = 0.0139\, U_{UP} + 0.0498\, U_N + 0.9973\, U_E$$

Inversion

General solution of the linear inverse problem:

$$\mathbf{m}^{est} = \mathbf{G}^{-g}\mathbf{d}^{obs}$$



R

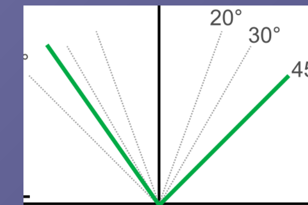
G is decomposed using a SVD.

To construct the general inverse we truncate the initial decomposition by taking into account only the eigenvalues containing information.

The general inverse can be written as follow (Menke, 1989):

$$\mathbf{m}^{est} = \overbrace{\mathbf{V}_p \Lambda_p^{-1} \mathbf{U}_p^T}^{\mathbf{G}^{-g}} \mathbf{d}^{obs}$$

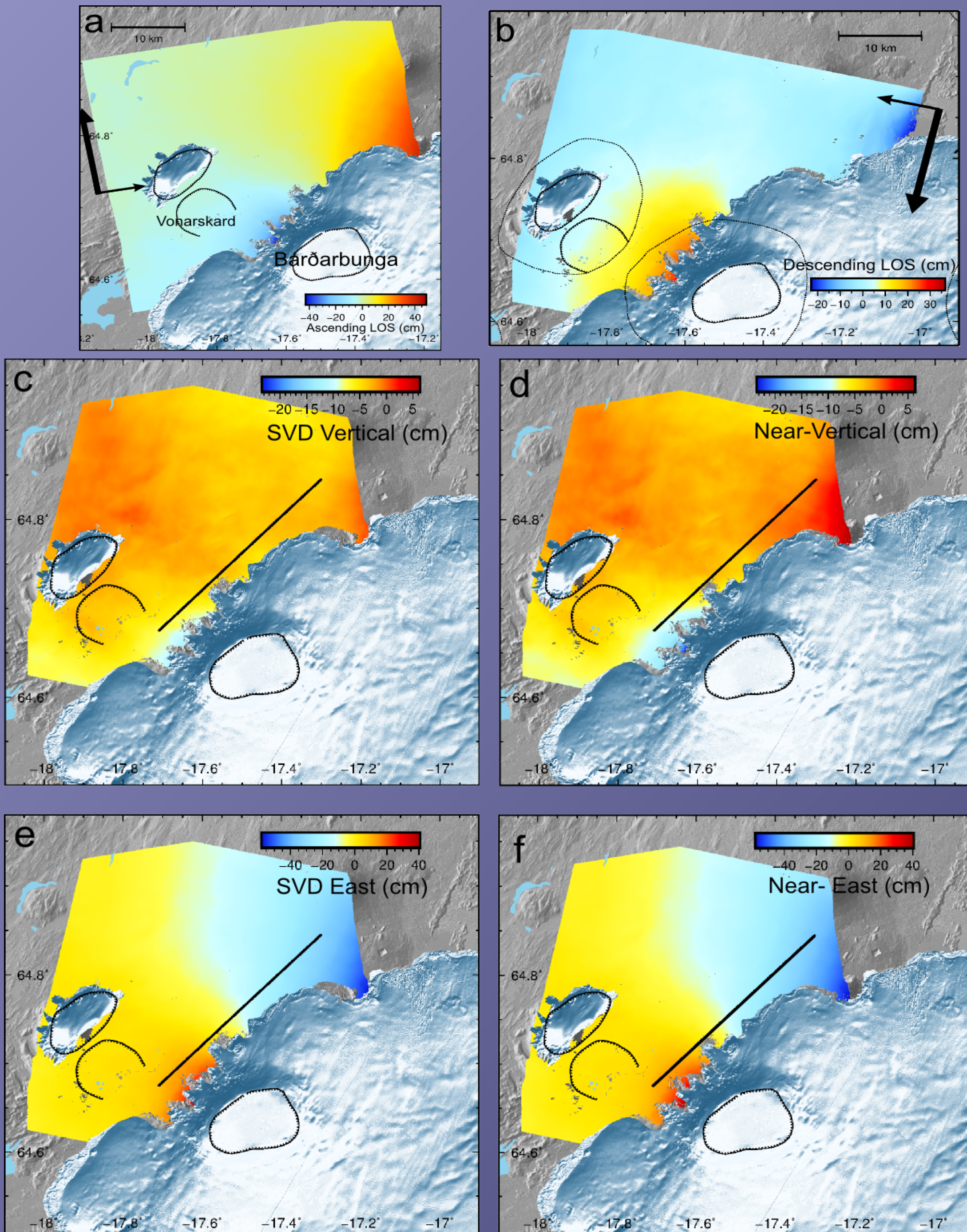
Bárðarbunga



Interferograms

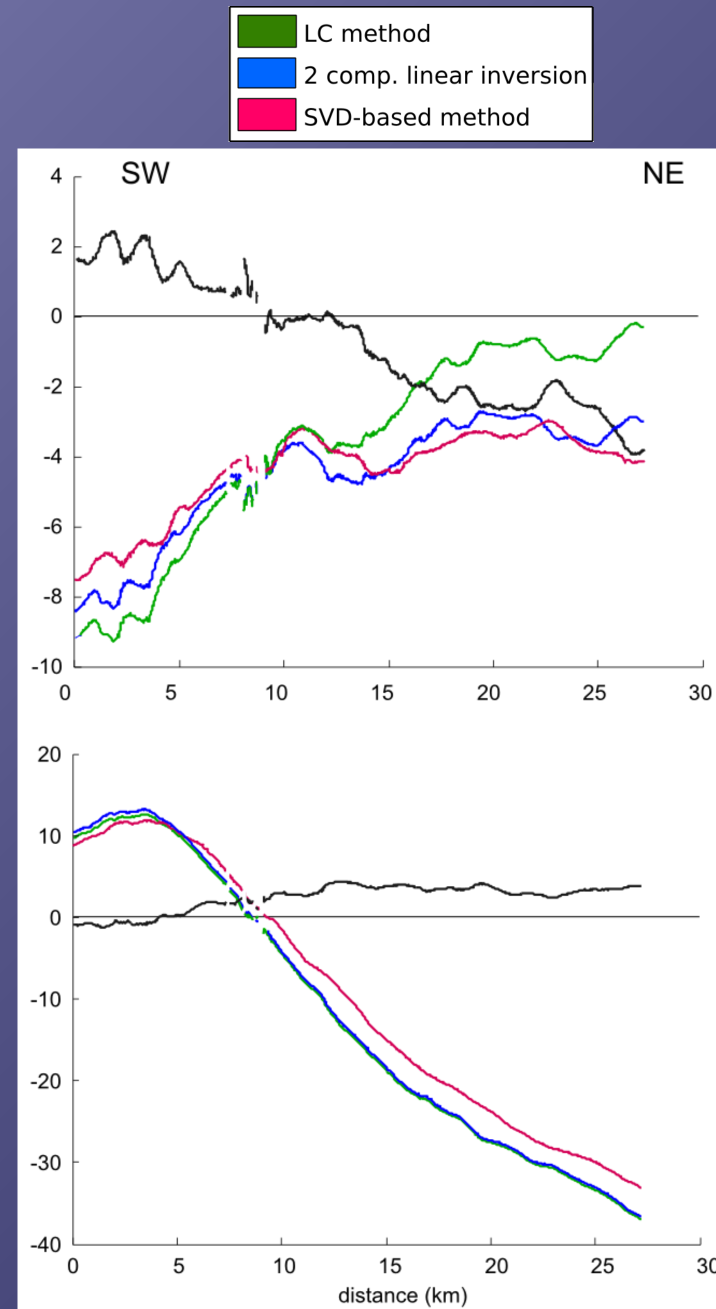
Retrieved Vertical component

Retrieved East component



Vertical component

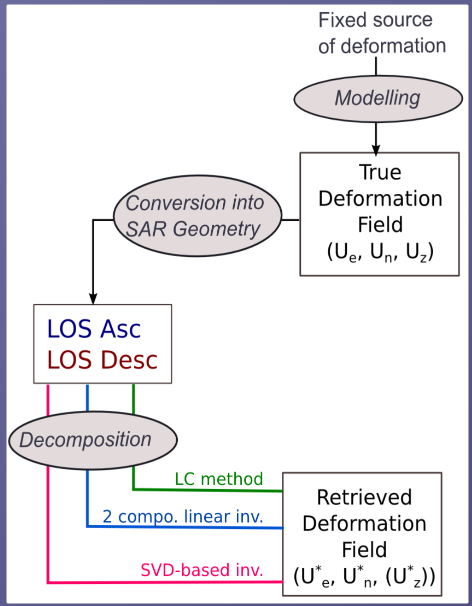
East component

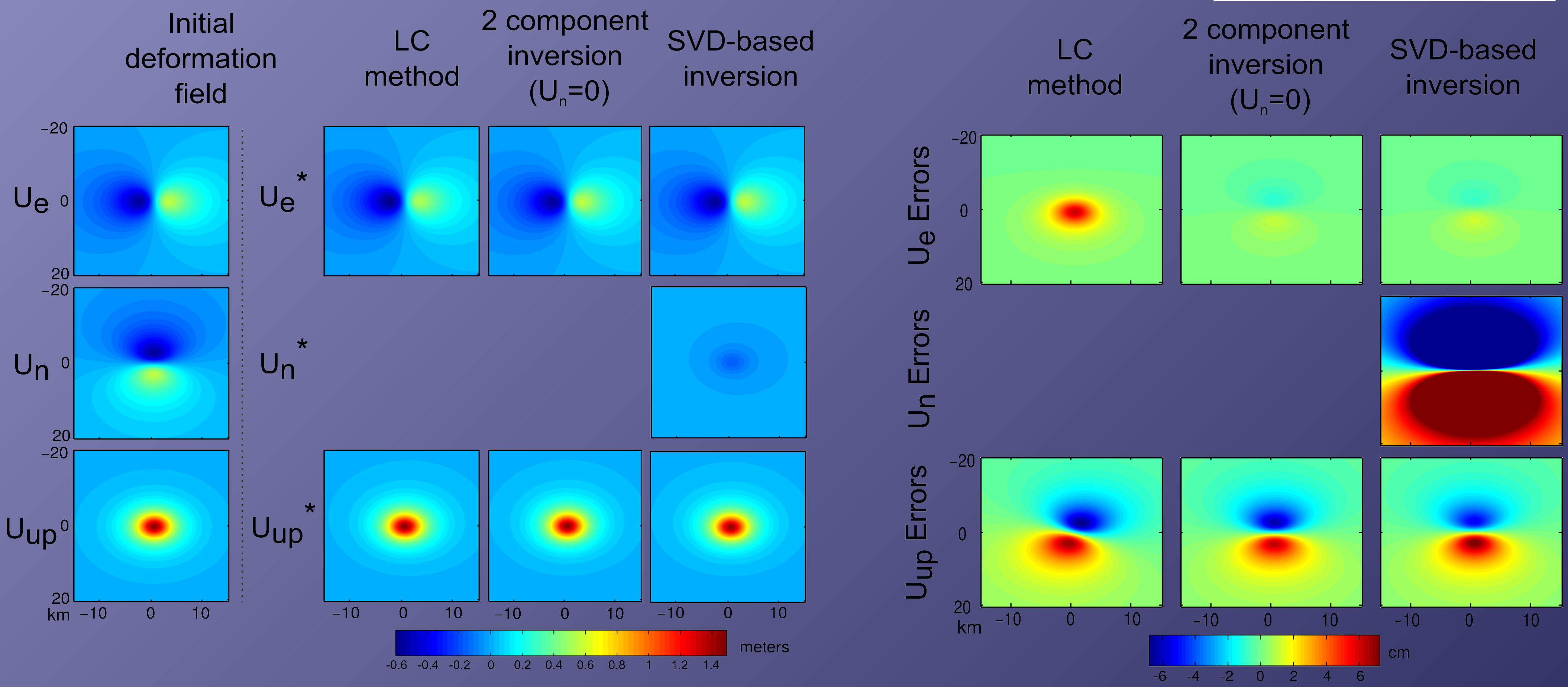


Comparison using Simulations

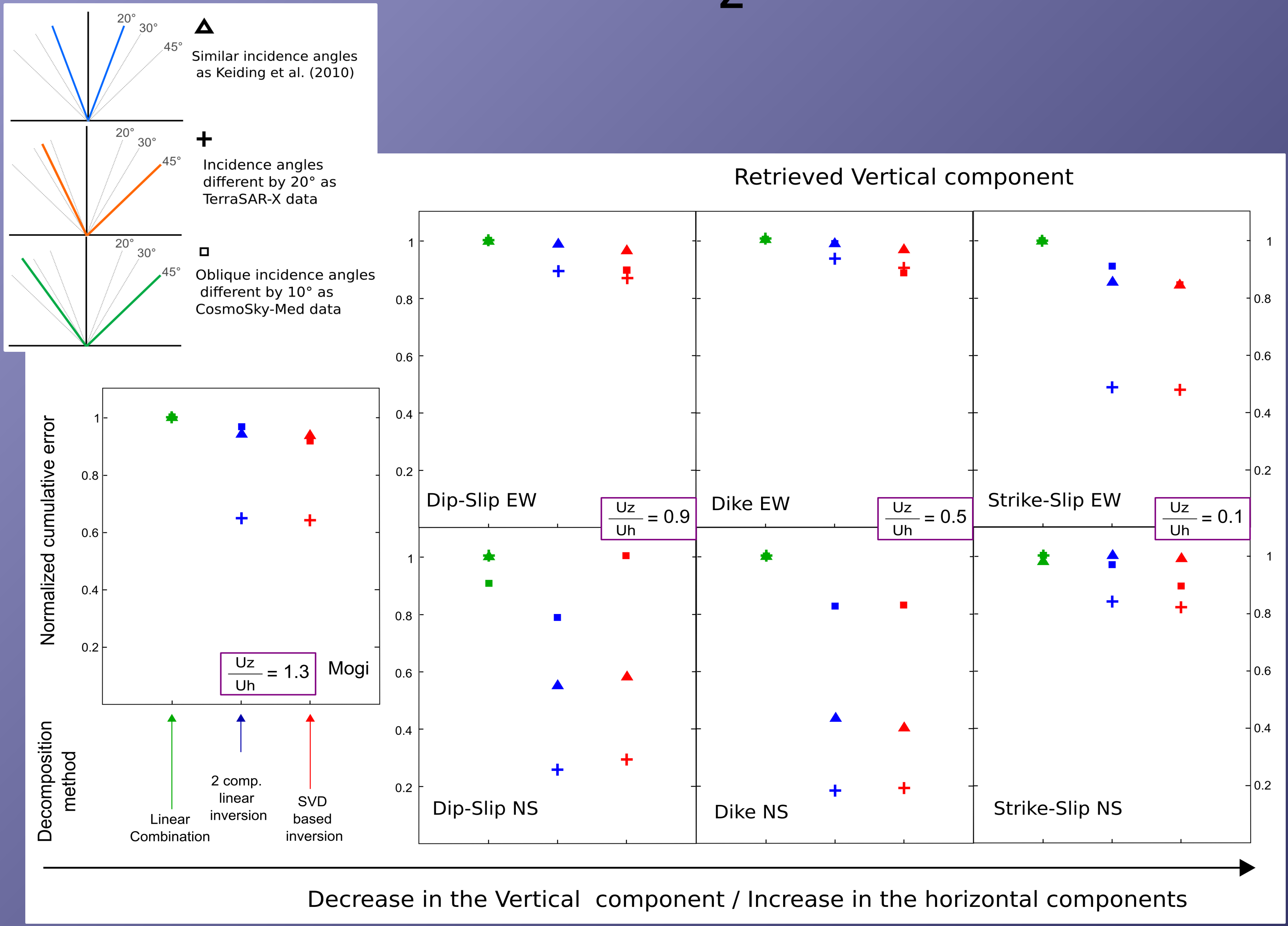
Mogi source: $dV = 0.1\ km^3$

$Z = 4\ km$

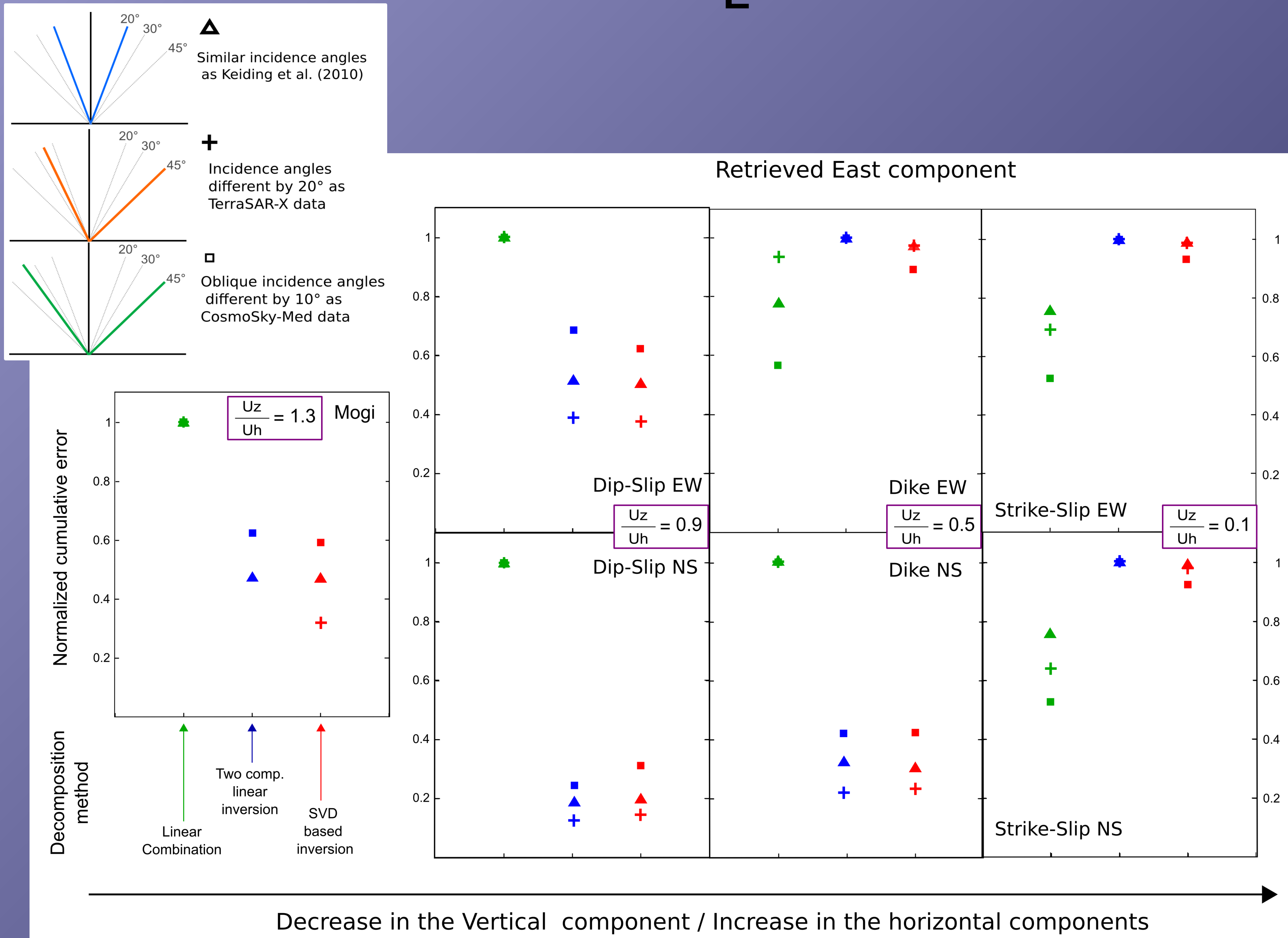




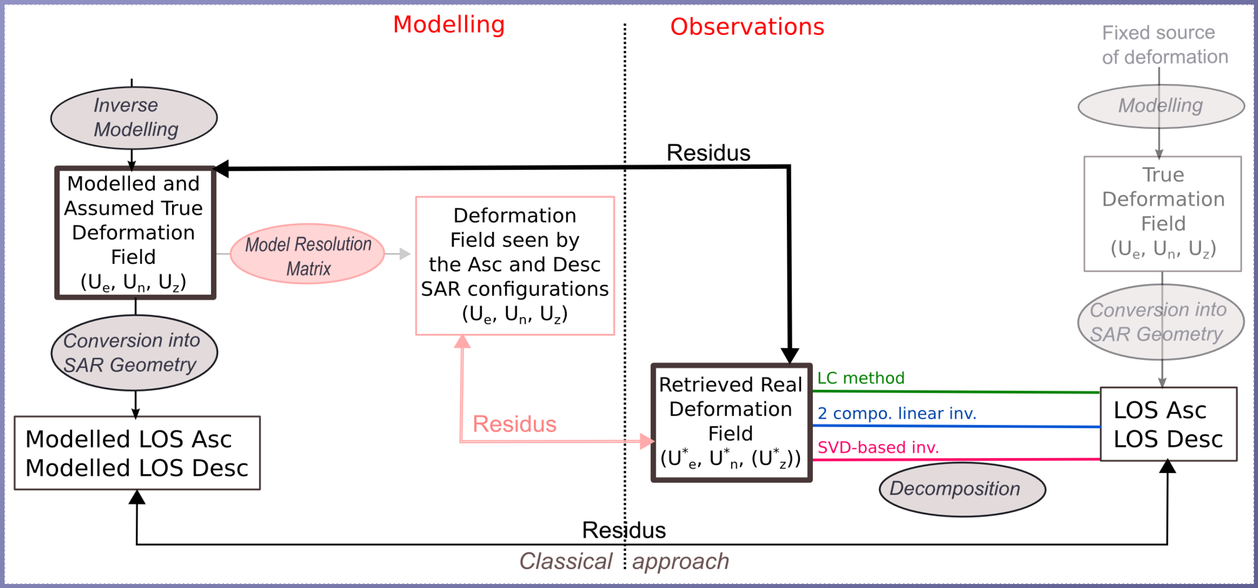
Detailed comparison: U_z



Detailed comparison: U_E



Comparison using simple Modelling



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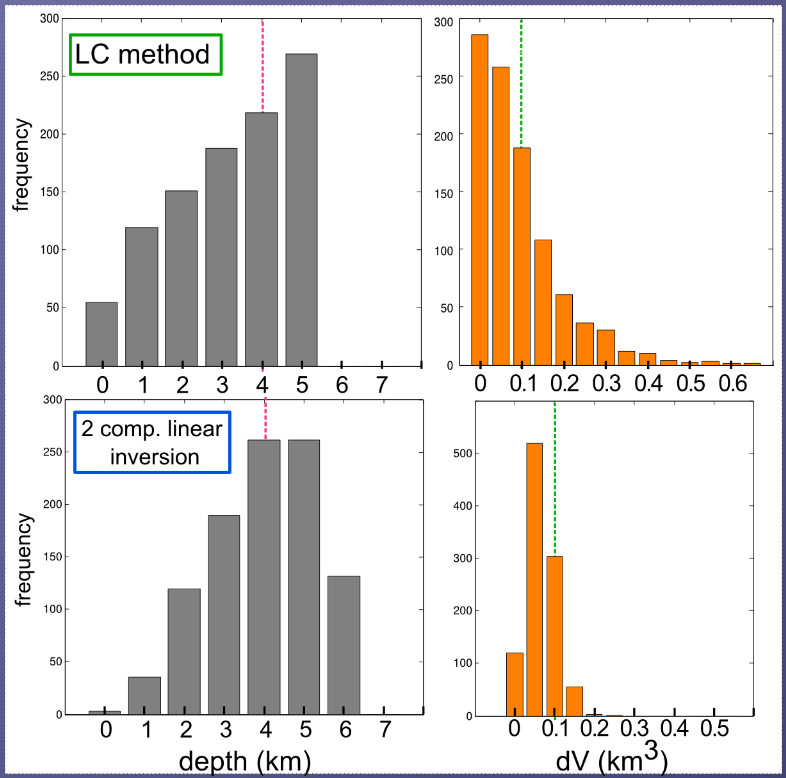
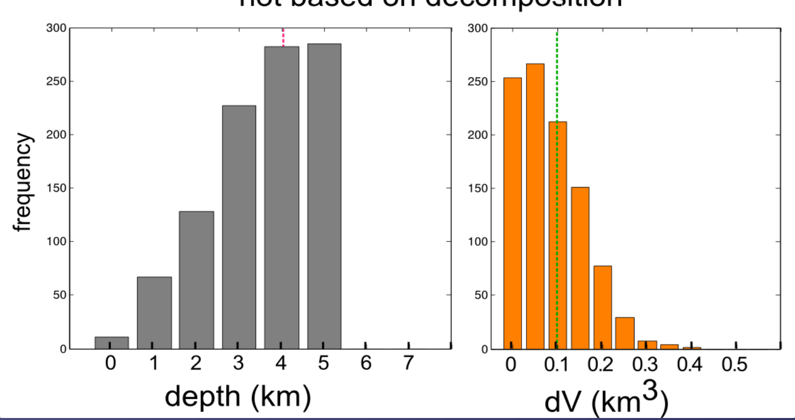
Modelling based on Decomposition methods



Classical inversion: not based on decomposition



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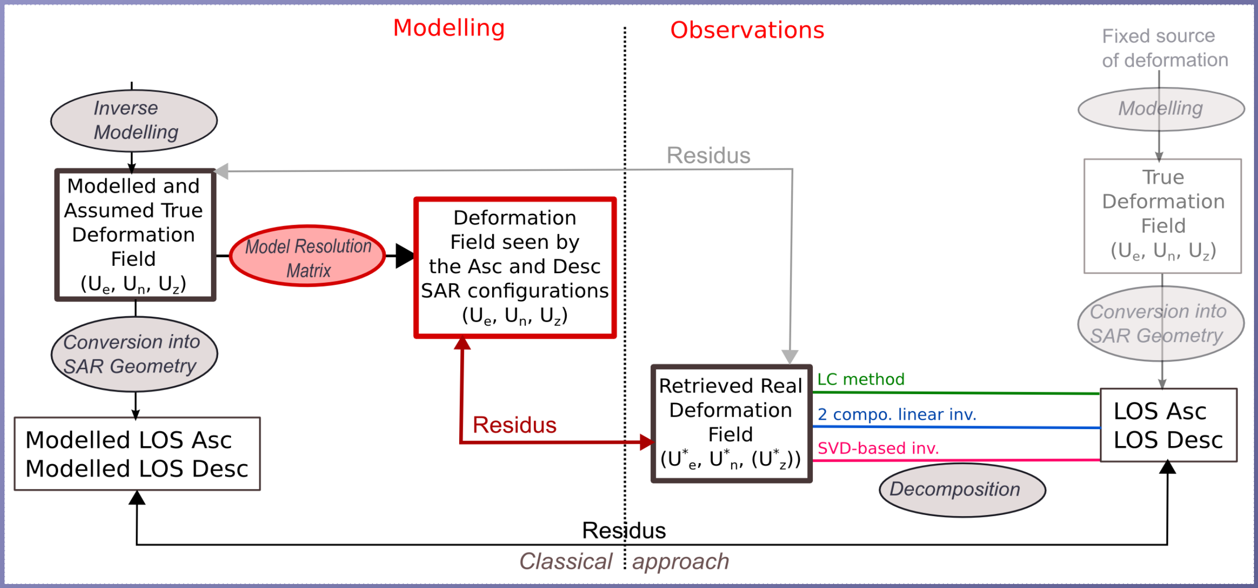

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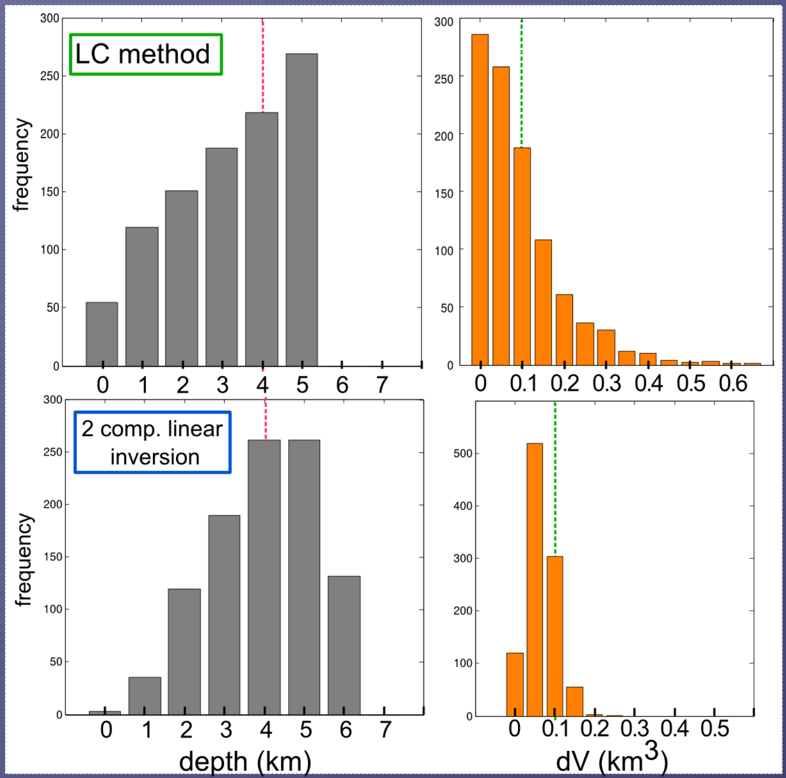

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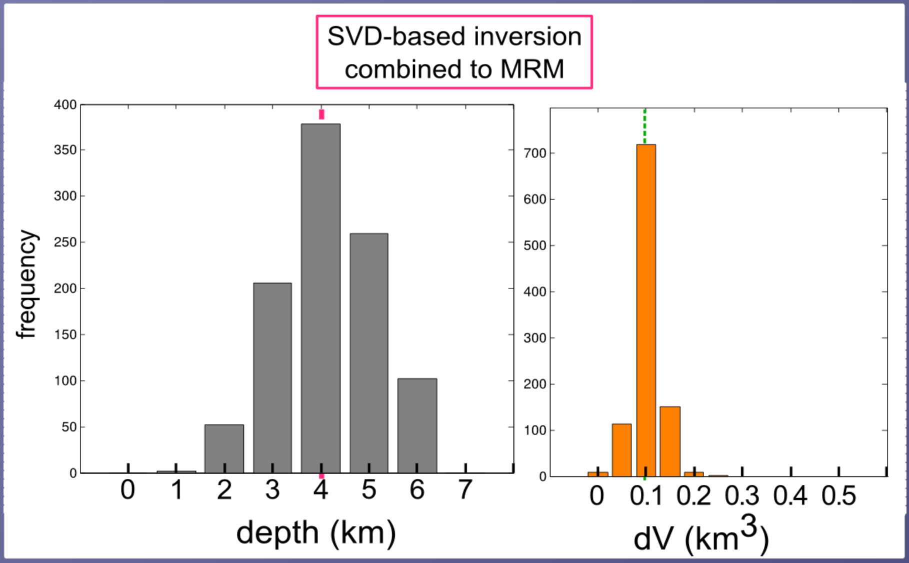
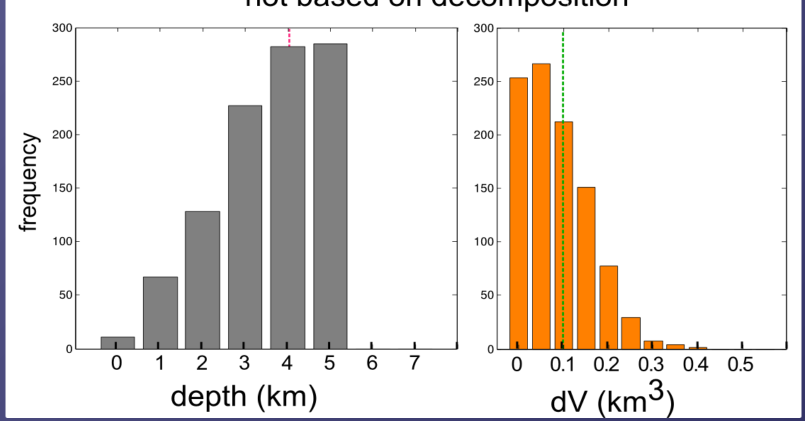


Classical inversion:
not based on decomposition



Summary

- Decomposition results will depend on: the combined viewing geometries, the deformation field and the orientation of its source.
- Mixed incidence angles, contributes to reduce errors on the reconstructed east and vertical components
- LC method: not particularly recommended
- Model resolution matrix quantifies the uncertainties on the true deformation field measurement and can be used for better constraining models.