TOWARDS THE ASSIMILATION OF DEFORMATION DATA IN VOLCANOLOGY

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OUTLINE OF THE TALK

- **Introduction**
  - Data assimilation: *the Ensemble Kalman Filter (EnKF)*
  - Reverso et al's *two-chamber model*
- **Synthetic Case**
  - State-parameter estimation
- **Discussion**
  - Using GNSS vs. InSAR data
  - Joint assimilation of GNSS and InSAR data
  - Comparison of EnKF with bayesian-based inversion (MCMC)
- **Key Points**

References:

[1] Bato MG, Pinel V, Yan Yajing (2017) "Assimilation of Deformation Data for Eruption Forecasting: Potentiality Assessment Based on Synthetic Cases". Front. Earth Sci. 5:48. doi: 10.3389/feart.2017.00048

[2] Bato MG, Pinel V, Yan Yajing (2016) "Volcano Deformation and Eruption Forecasting using Data Assimilation: Building the Strategy". AGU Fall Meeting, San Francisco, USA

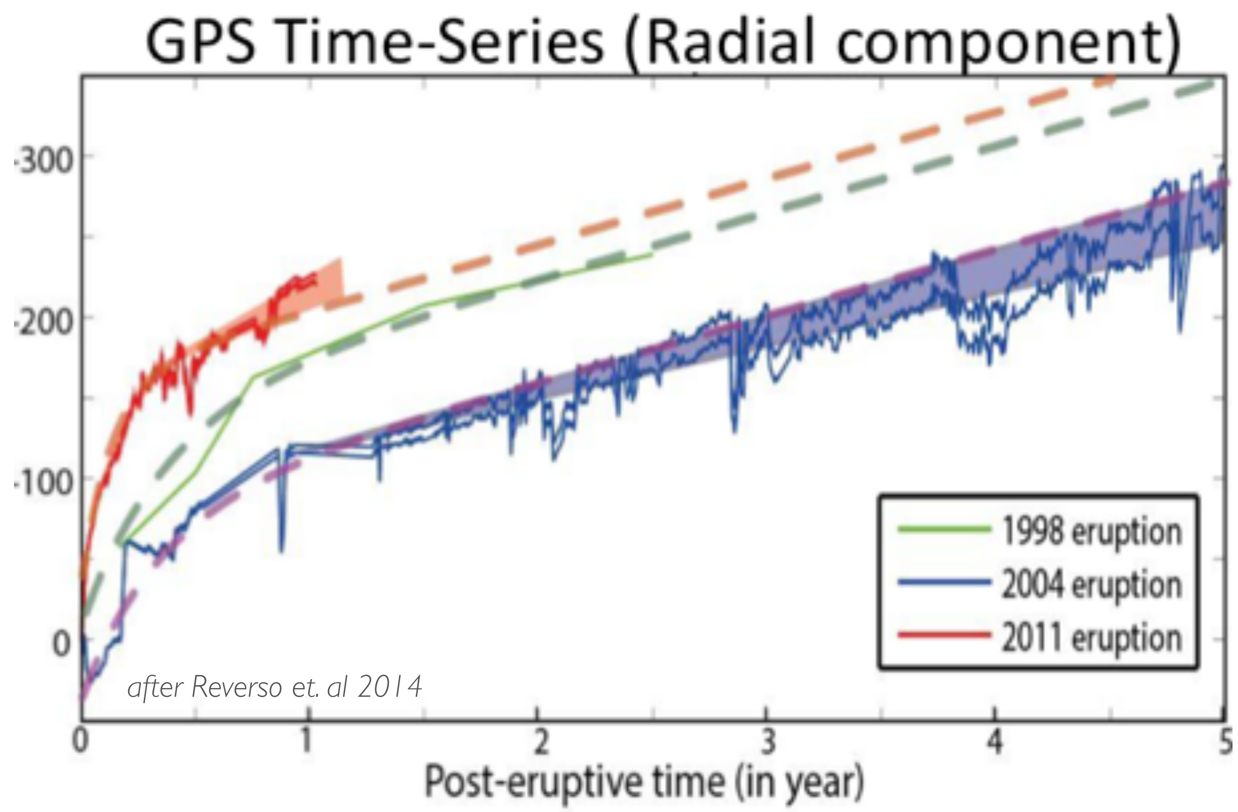
Press Mentions:

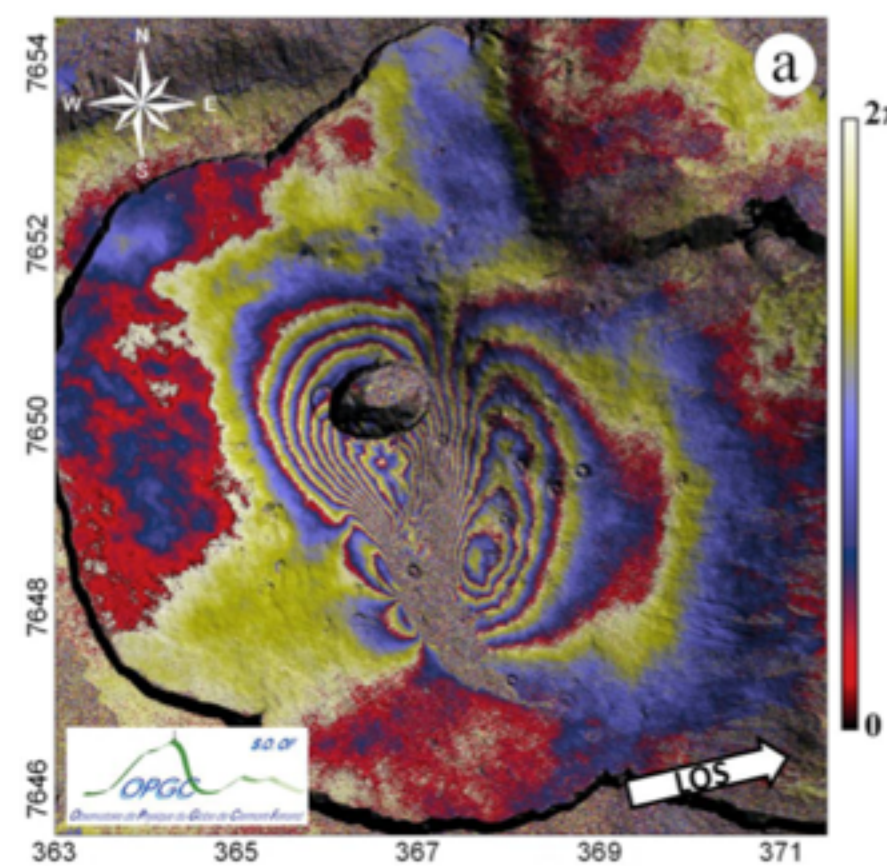
[1] Volcano Forecast? New Technique Could Better Predict Eruptions, Scientific American
[2] Scientists are trying to use satellites to forecast volcanic eruptions, CNBC
[3] Think weather forecasts are bad? Try forecasting a volcanic eruption, Popular Science
[4] Predicting eruptions using satellites and math, Eurekalert
[5] Scientists predict volcanic eruptions with satellites and GPS, CNN Tech

HOW DO WE KNOW IF A VOLCANO IS ABOUT TO ERUPT?

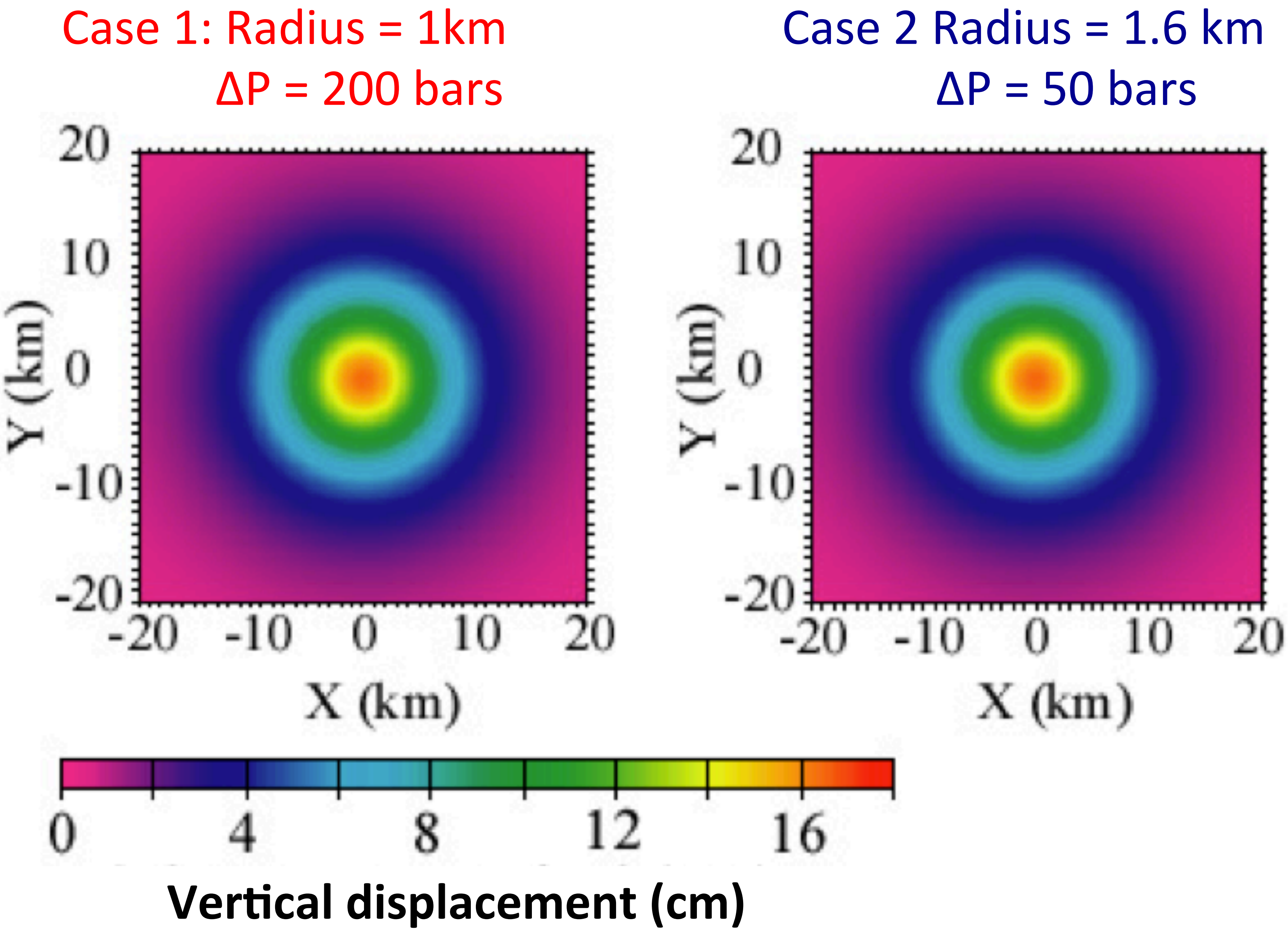
Key parameter: Overpressure

As the pressure accumulates, the volcano becomes unstable. Volcanic eruption can occur if it surpasses a critical threshold overpressure.





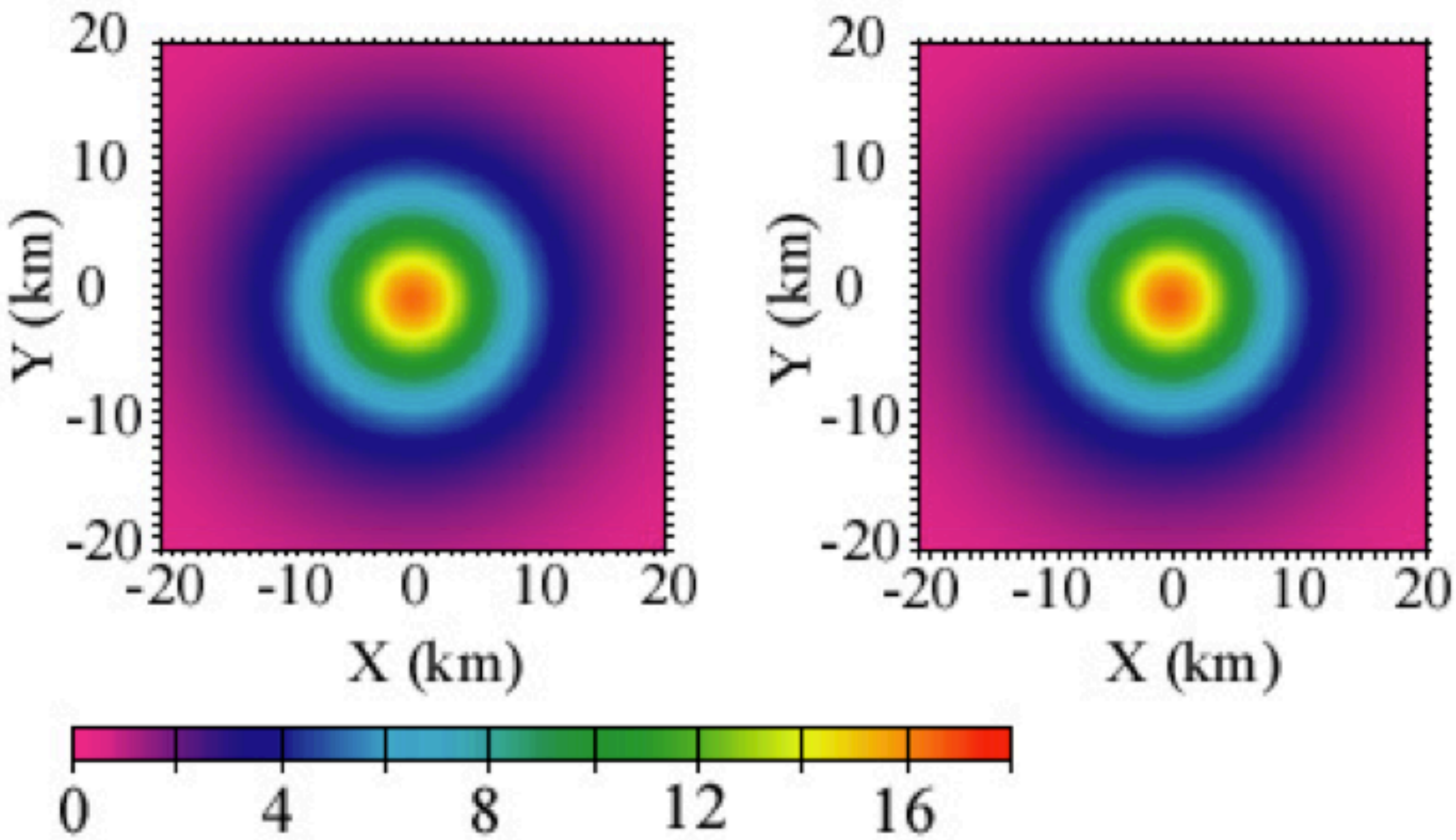
Bato et. al. 2016



Case 1: Radius = 1km
ΔP = 200 bars
Case 2 Radius = 1.6 km
ΔP = 50 bars
20
10
0
-10
-20
Y (km)
-20 -10 0 10 20
X (km)
0 4 8 12 16
Vertical displacement (cm)

Case 1: Radius = 1km
ΔP = 200 bars

Case 2 Radius = 1.6 km
ΔP = 50 bars

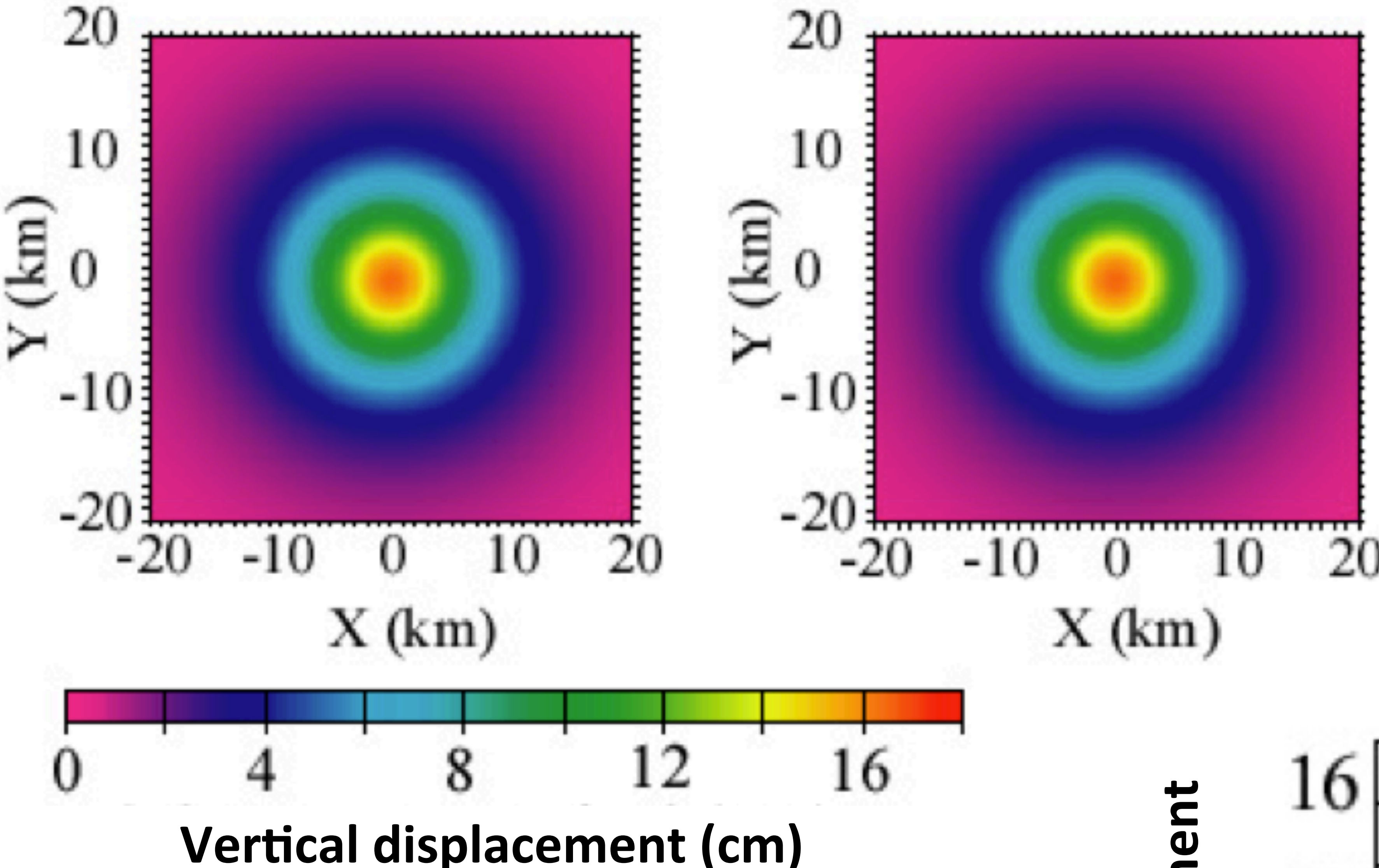


Vertical displacement (cm)

Almost no difference

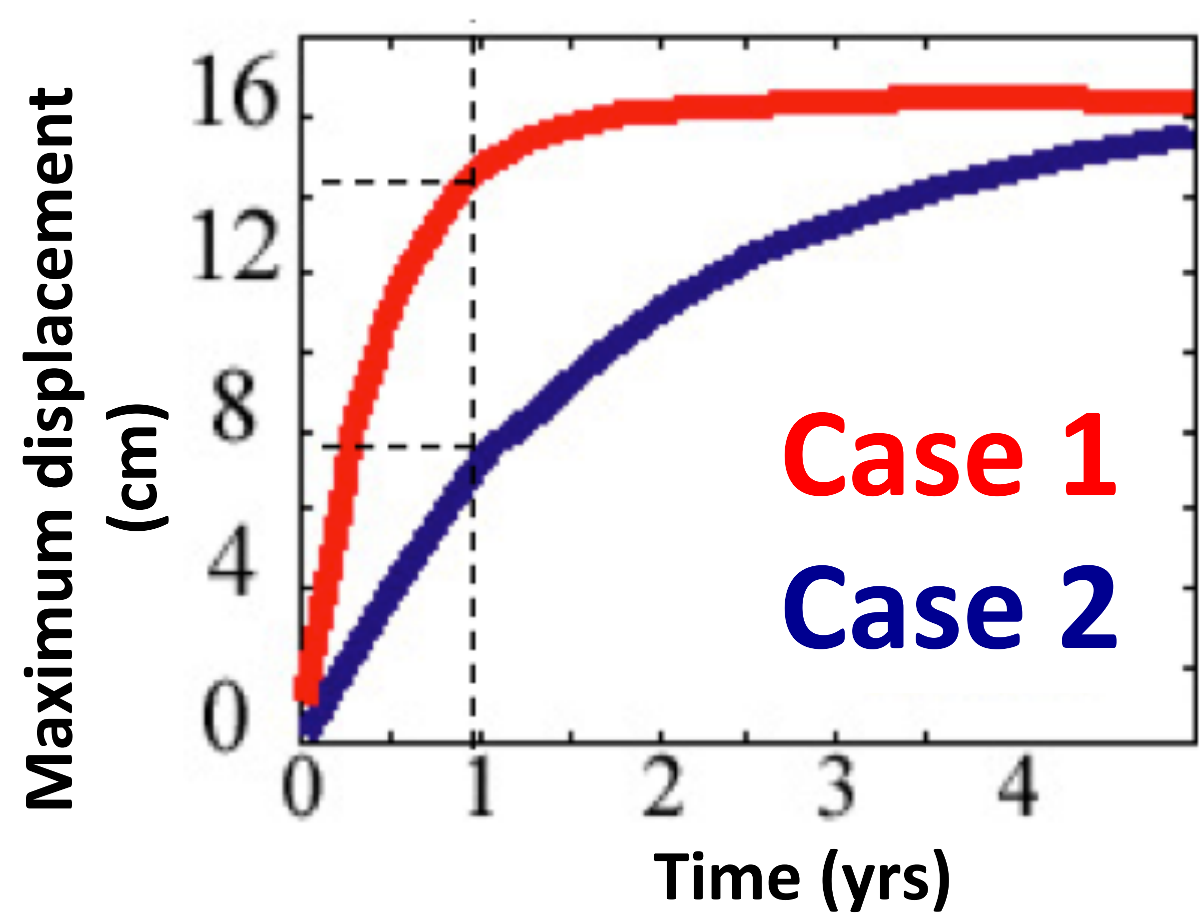
Case 1: Radius = 1km
ΔP = 200 bars

Case 2 Radius = 1.6 km
ΔP = 50 bars



Almost no difference

A smaller reservoir is easily filled with magma, hence, it is pressurized quickly.



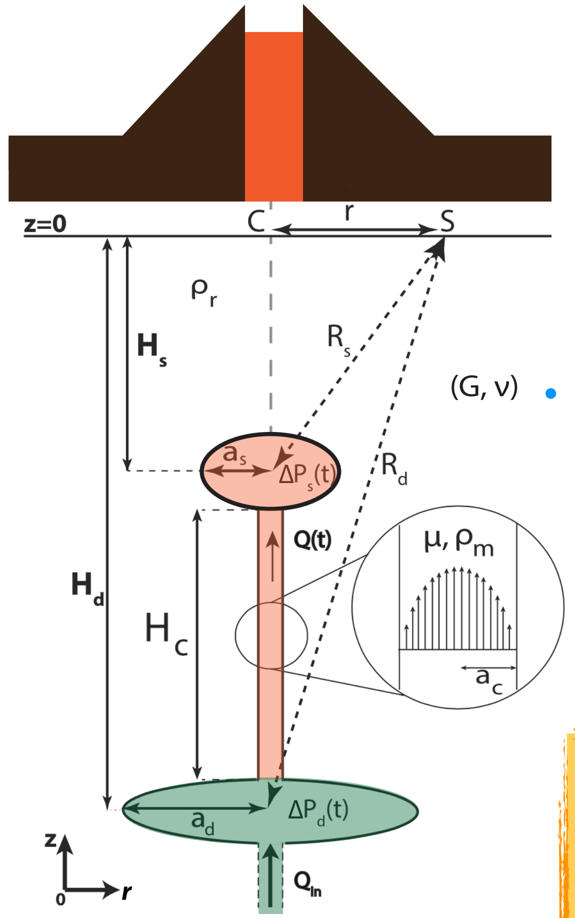
DATA ASSIMILATION: GENERAL IDEA

- Models are incorporated by errors
  - $x_{t+1} = \mathcal{M} x_t + q$
- Observations are not free of noise
  - $D = \mathcal{H} x^{\dagger} + \epsilon$
- Model-data fusion technique

DATA ASSIMILATION: GENERAL IDEA

Data assimilation is a sequential time-forward process that best combines models and observations, sometimes a priori information based on error statistics, to predict the state of a dynamical system.

DATA ASSIMILATION: GENERAL IDEA



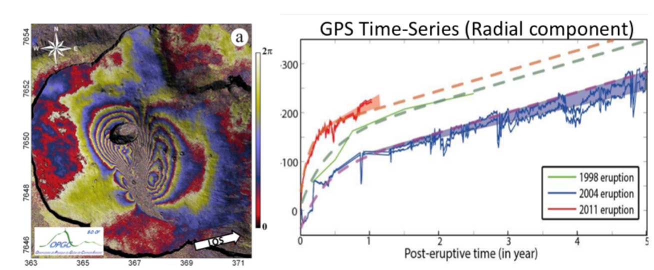
(G, v)

$$x^f = \begin{bmatrix} \Delta P_s \\ \Delta P_d \\ a_d \\ Q_{in} \end{bmatrix}$$

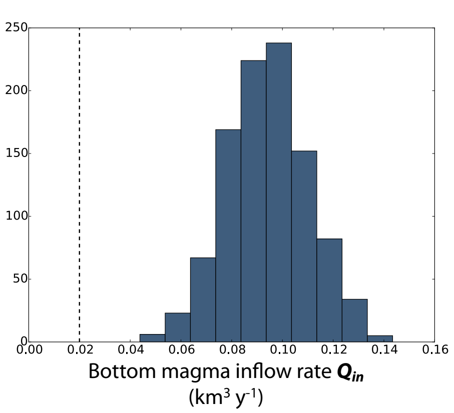
Assimilation Scheme

Observations

$$D = \mathcal{H}x^\dagger + \epsilon$$



A priori information



Dynamical model

$$x^f_{t+1} = \mathcal{M}x^a_t + q$$

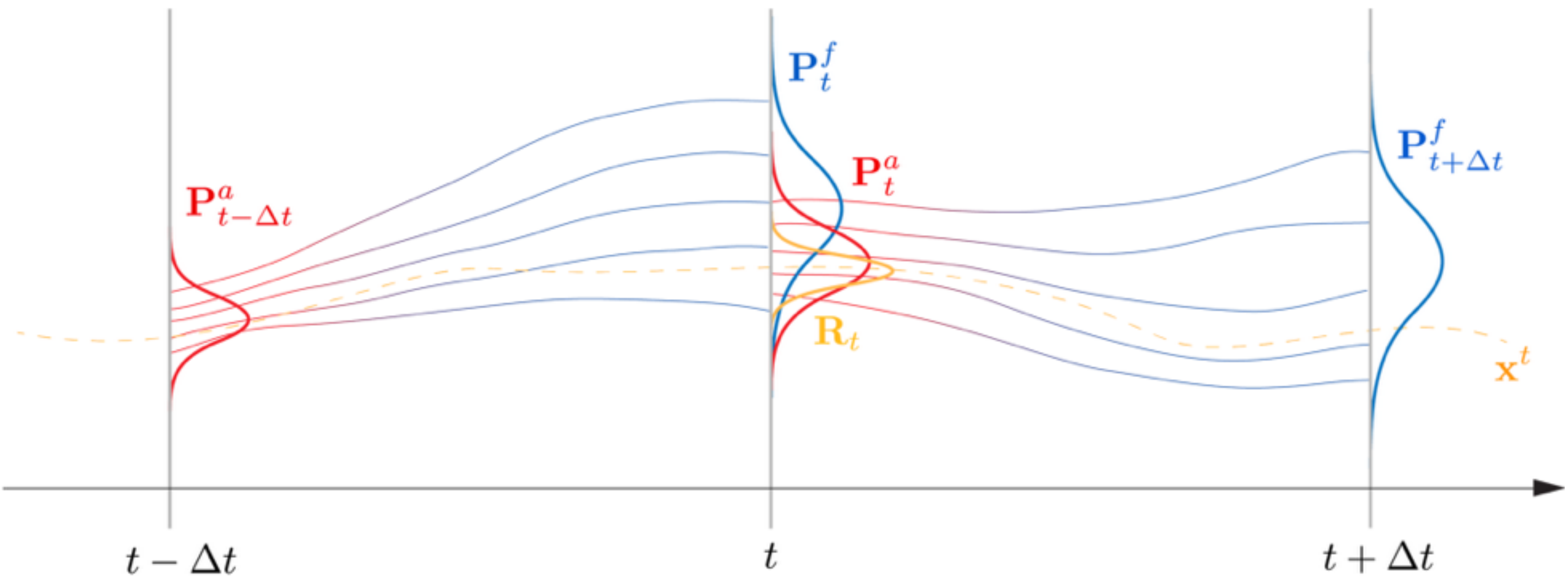
$$x^a$$

ASSIMILATION SCHEME: ENSEMBLE KALMAN FILTER

Model Error Covariance

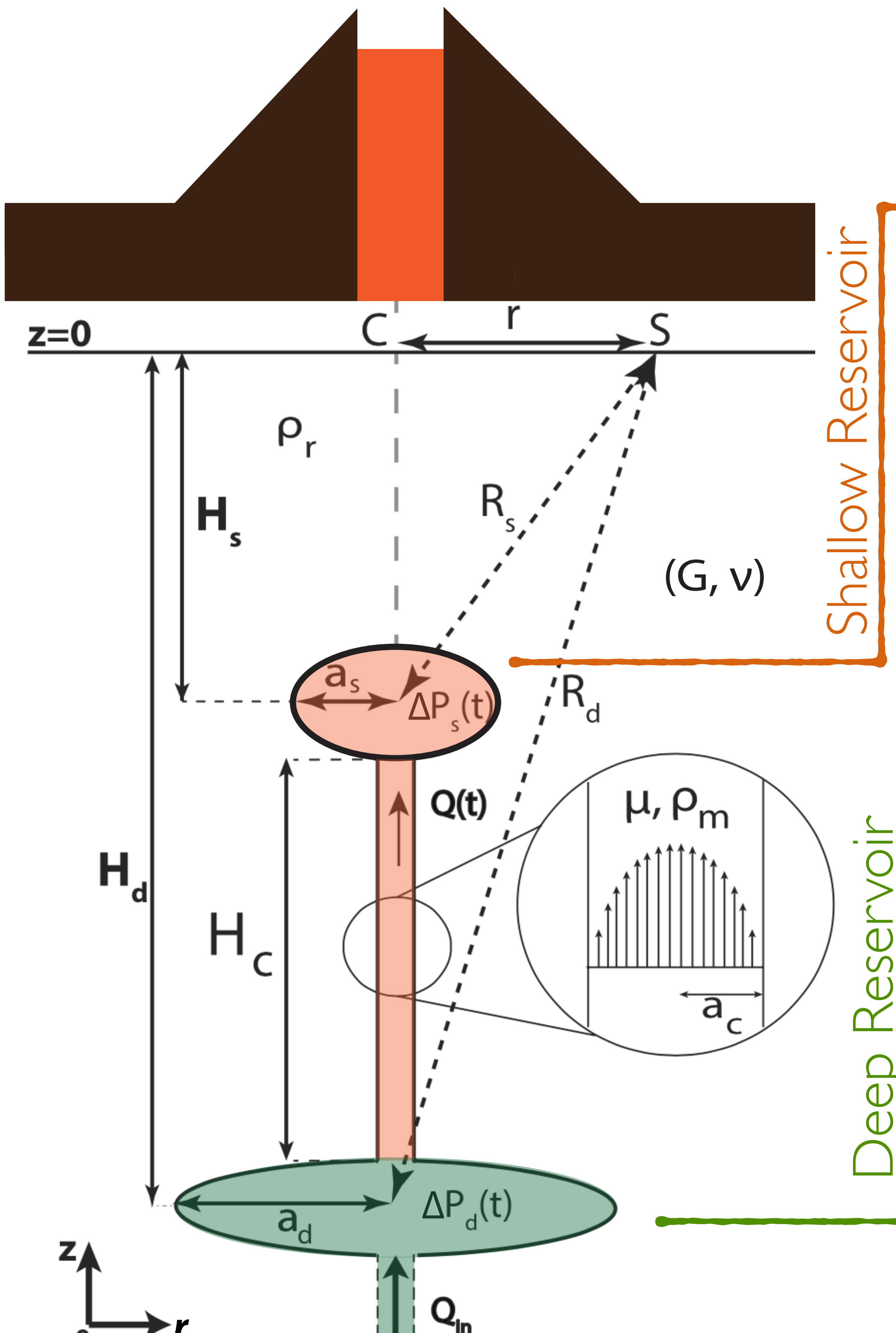
$$P^f = \overline{(x^f - \overline{x^f})(x^f - \overline{x^f})^T}$$

$$P^a = \overline{(x^a - \overline{x^a})(x^a - \overline{x^a})^T}$$



Sanchez, 2016

THE TWO-CHAMBER MODEL



modified after Reverso et. al. 2014

Overpressures

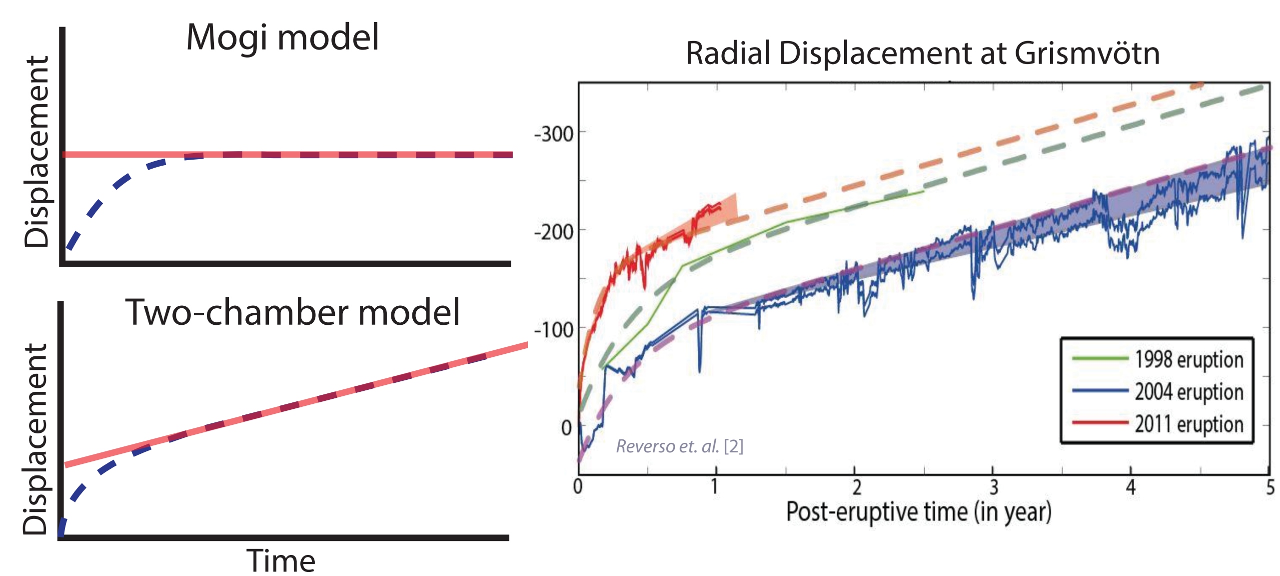
$$\frac{\Delta P_{s_{t_{i+1}}} - \Delta P_{s_{t_i}}}{t_{i+1} - t_i} = \frac{G a_c^4}{8\mu\gamma_s H_c a_s^3}((\rho_r - \rho_m) g H_c + \Delta P_{d_{t_i}} - \Delta P_{s_{t_i}})$$

$$\frac{\Delta P_{d_{t_{i+1}}} - \Delta P_{d_{t_i}}}{t_{i+1} - t_i} = \frac{G}{\gamma_d \pi a_d^3} Q_{in} - \frac{\gamma_s a_s^3}{\gamma_d a_d^3} \frac{\Delta P_{s_{t_{i+1}}} - \Delta P_{s_{t_i}}}{t_{i+1} - t_i}$$

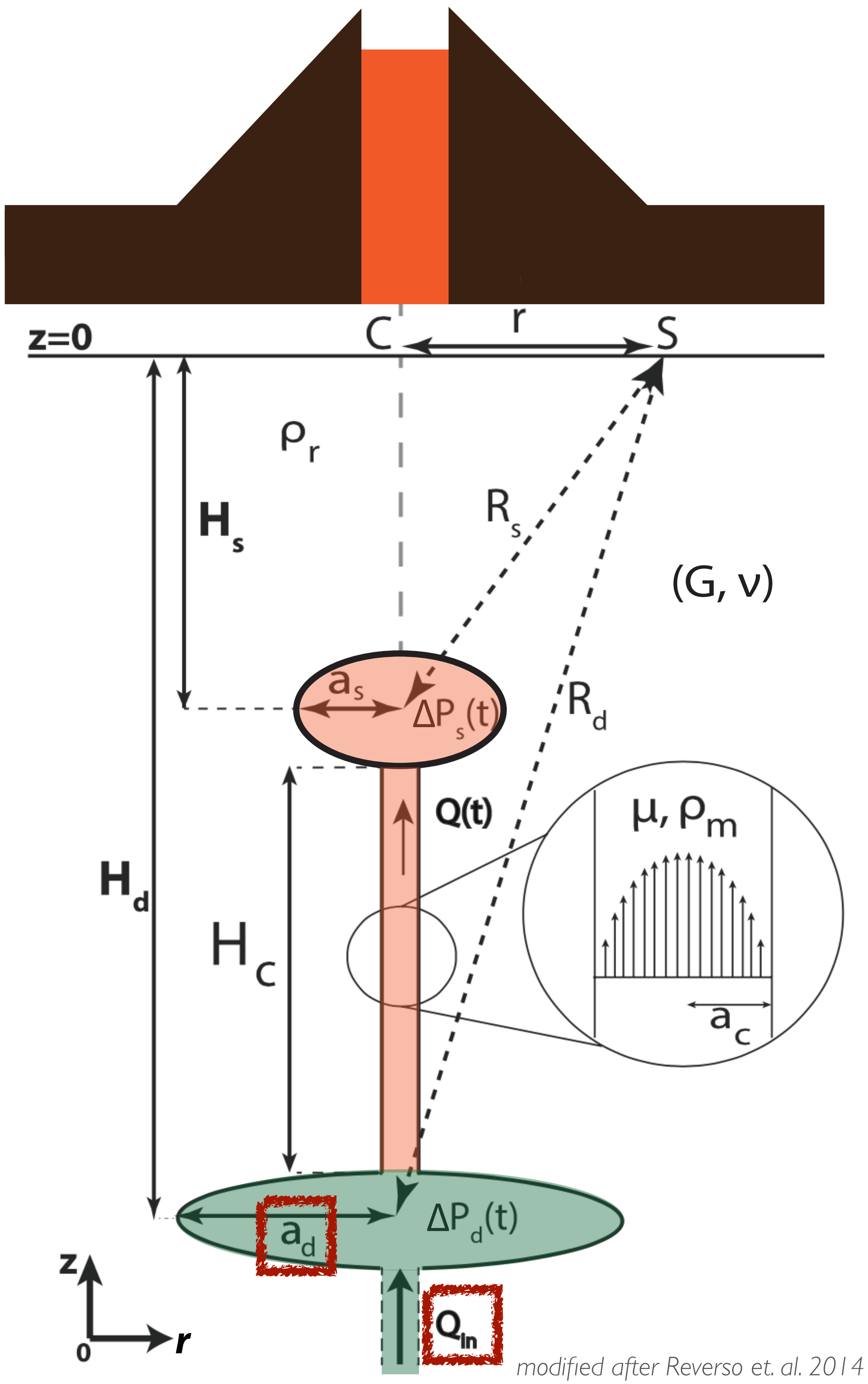
Overpressure-Displacement Relationship

$$u_R(r, t_i) = \frac{(1-v)}{G} r \left(\alpha_s \frac{a_s^3}{R_s^3} \Delta P s_{t_i} + \alpha_d \frac{a_d^3}{R_d^3} \Delta P_{d_{t_i}} \right)$$

$$u_z(r, t_i) = \frac{(1-v)}{G} \left(H_s \alpha_s \frac{a_s^3}{R_s^3} \Delta P s_{t_i} + H_d \alpha_d \frac{a_d^3}{R_d^3} \Delta P_{d_{t_i}} \right)$$



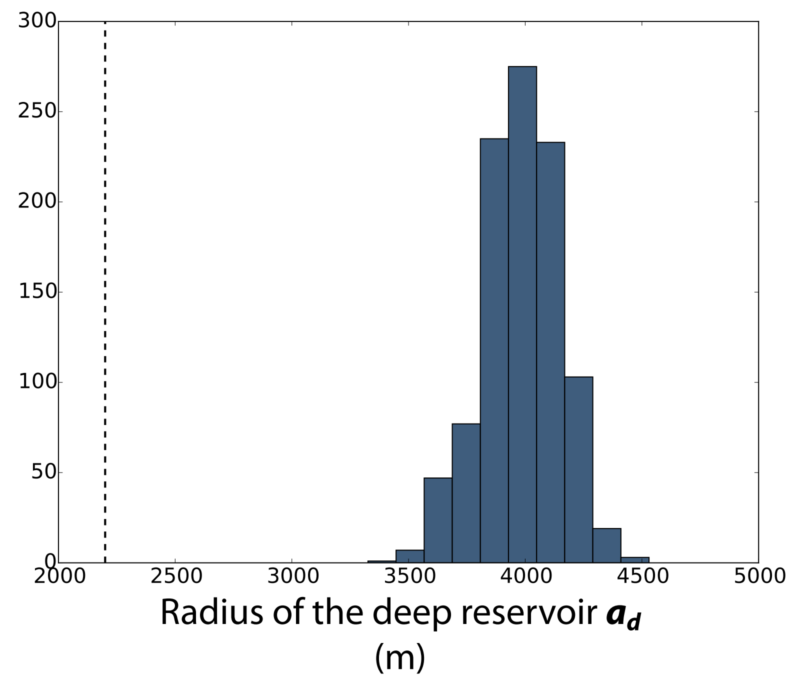
THE TWO-CHAMBER MODEL

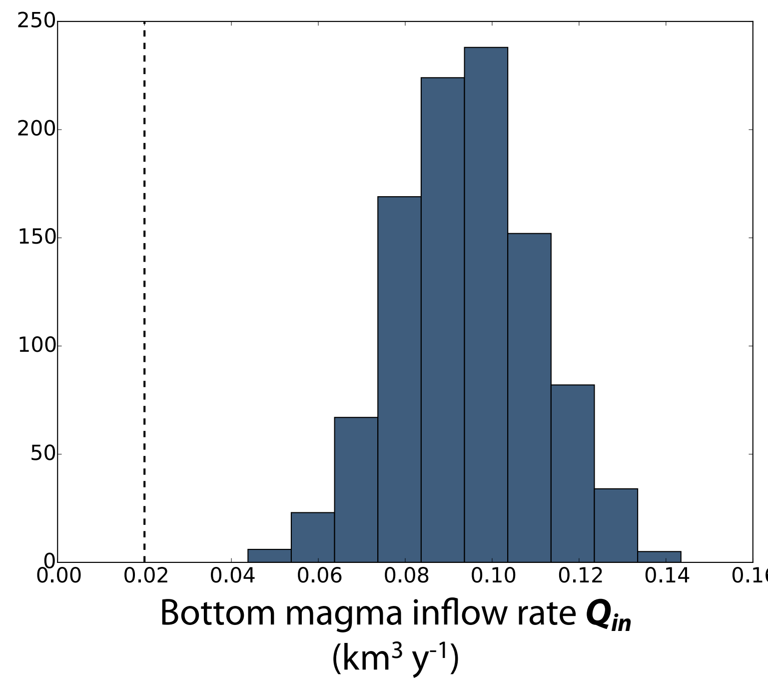


modified after Reverso et. al. 2014

For our assimilation scheme, 2 model parameters are fixed to be uncertain:

- ☑ The radius of the deep reservoir: $\boldsymbol{a_d}$
- ☑ Basal magma inflow rate: $\boldsymbol{Q_{in}}$

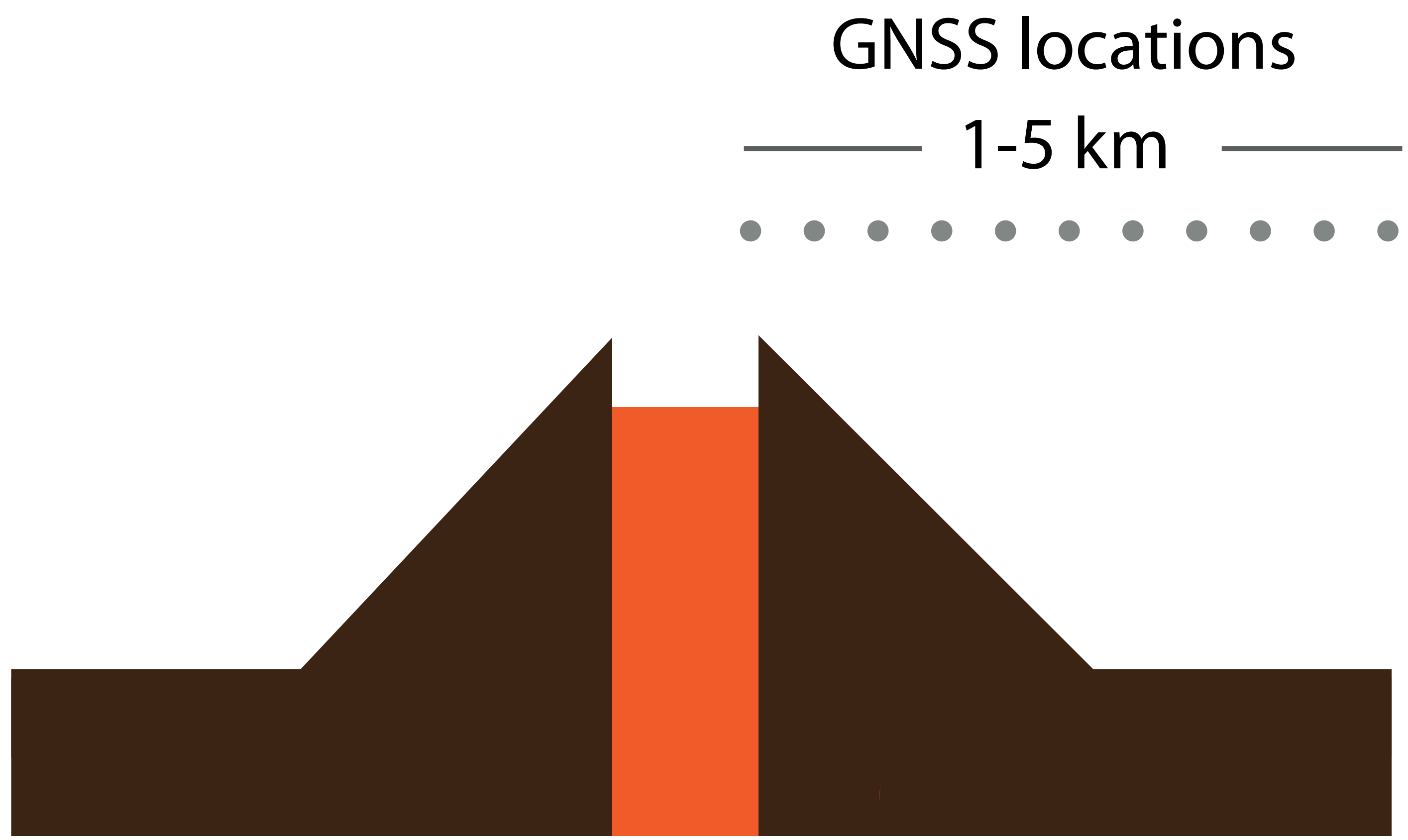




SYNTHETIC CASE

☑ State-Parameter Estimation (estimating the overpressures and the uncertain parameters)

SYNTHETIC CASE: STRATEGY



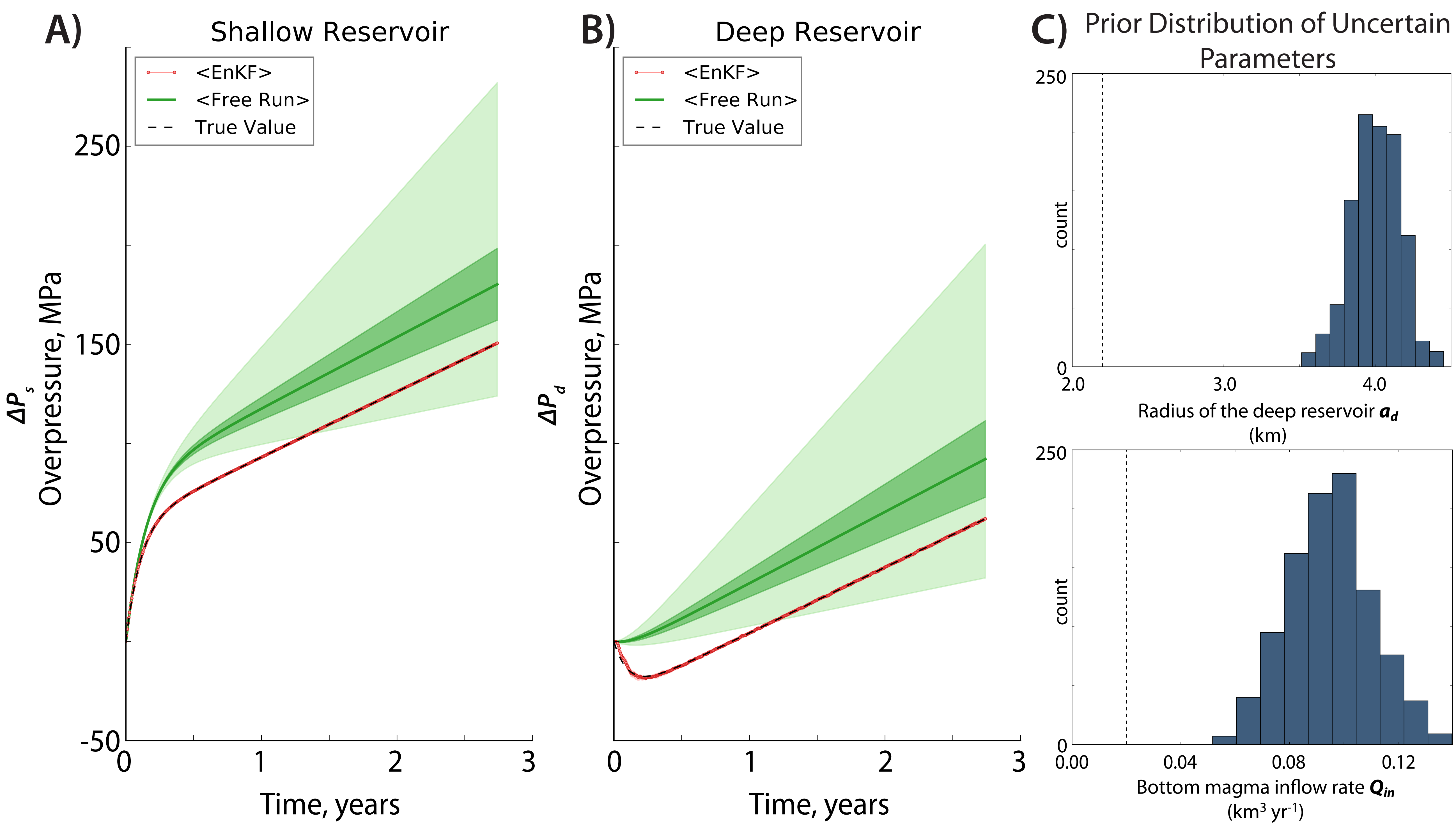
The assimilation interval, $\Delta t = 2 \text{ days}$

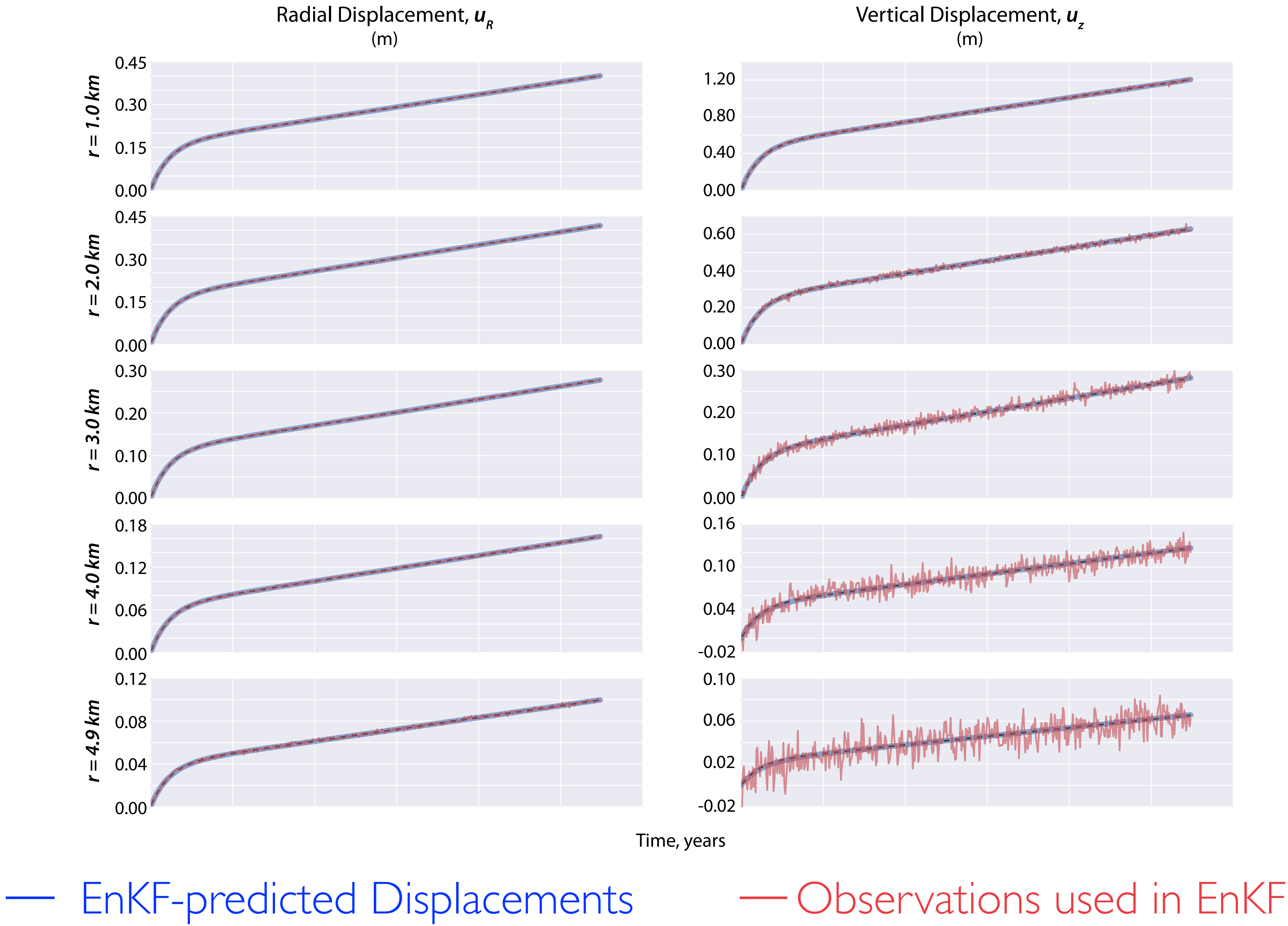
The frequency of available observation is also 2 days.

80 observations are used for the synthetic cases.

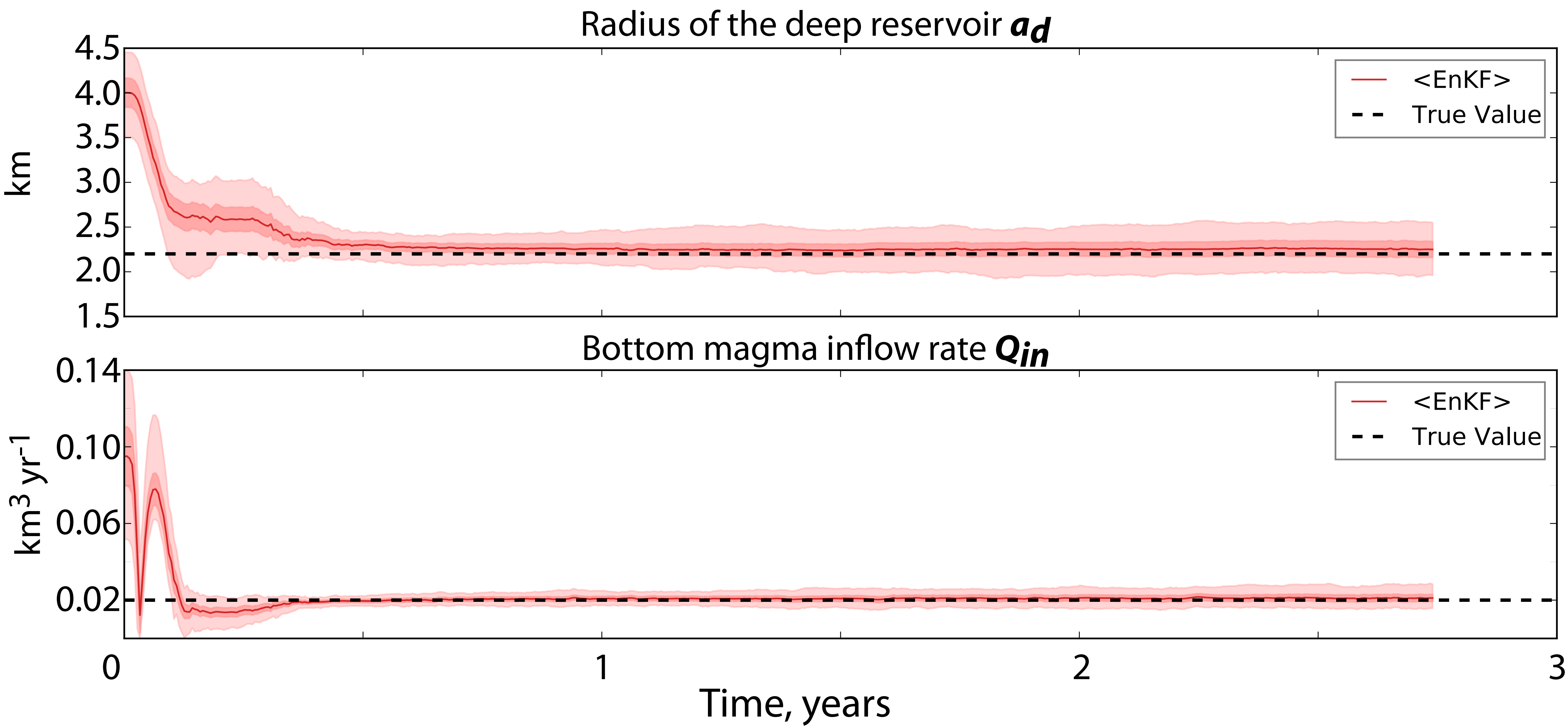
- ☑ 40 radial and 40 vertical

CASE: STATE-PARAMETER ESTIMATION





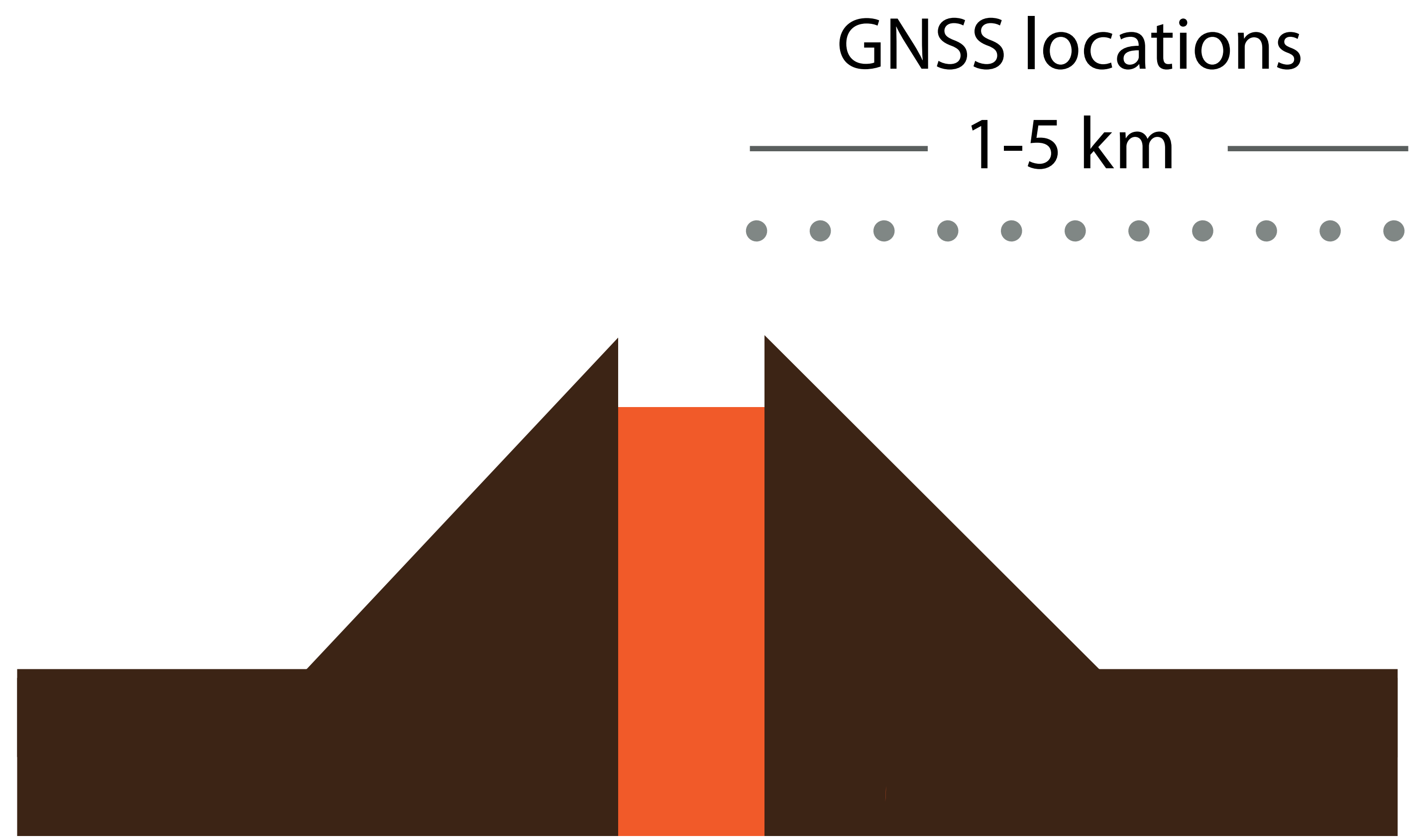
Radial Displacement, u_R
(m)
Vertical Displacement, u_z
(m)
r = 1.0 km
r = 2.0 km
r = 3.0 km
r = 4.0 km
r = 4.9 km
0.45
0.30
0.15
0.00
0.20
0.10
0.18
0.12
0.06
0.08
0.04
1.20
0.80
0.40
0.60
0.16
-0.02
0.02
Time, years
EnKF-predicted Displacements
Observations used in EnKF



Radius of the deep reservoir a_d
km
4.5
4.0
3.5
3.0
2.5
2.0
1.5
<EnKF>
True Value
Bottom magma inflow rate Q_{in}
$km^3\ yr^{-1}$
0.14
0.10
0.06
0.02
<EnKF>
True Value
0
1
2
3
Time, years

FURTHER DISCUSSIONS ON THE USE OF DEFORMATION DATA

GNSS VS. INSAR:
HOW **SPATIAL** RESOLUTION AFFECTS THE ASSIMILATION



GNSS dataset:

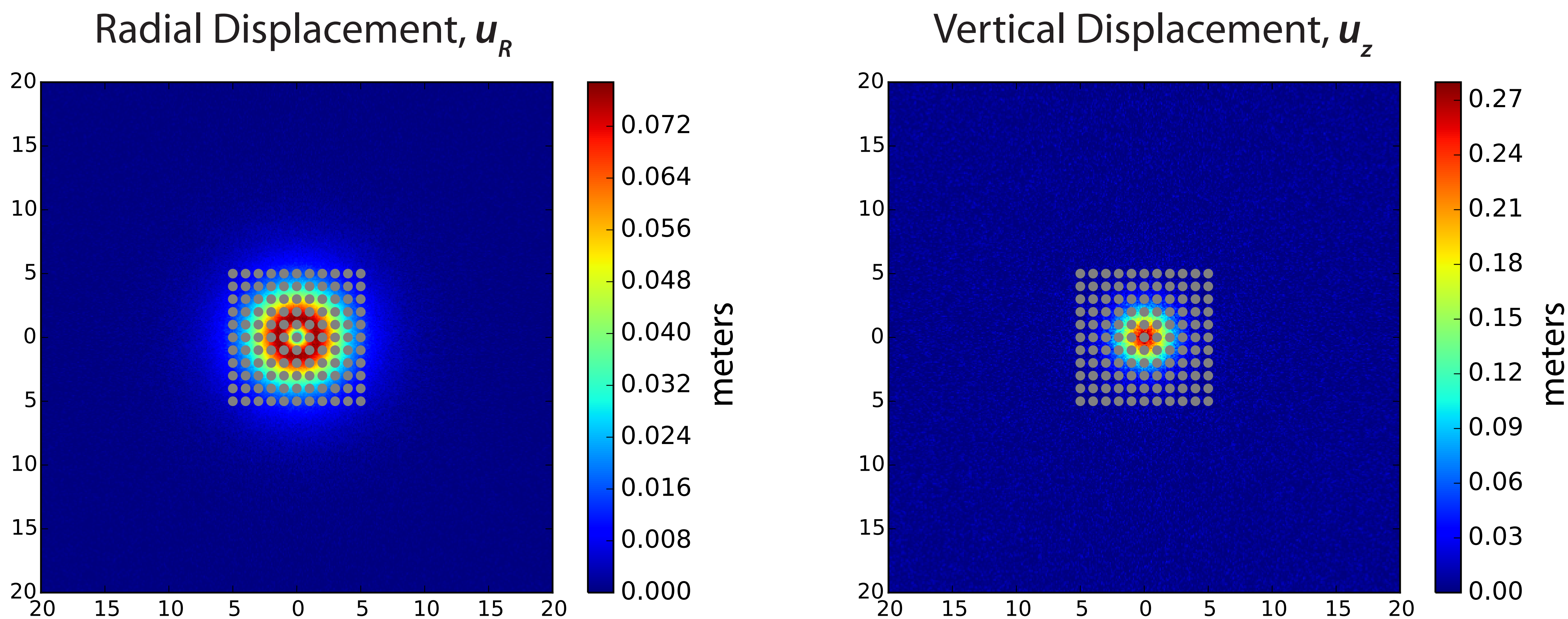
The assimilation interval, $\Delta t = 2 \text{ days}$

The frequency of available observation is also 2 days.

10 observations are used for the synthetic cases.

- ☑ 5 radial and 5 vertical

GNSS VS. INSAR:
HOW **SPATIAL** RESOLUTION AFFECTS THE ASSIMILATION



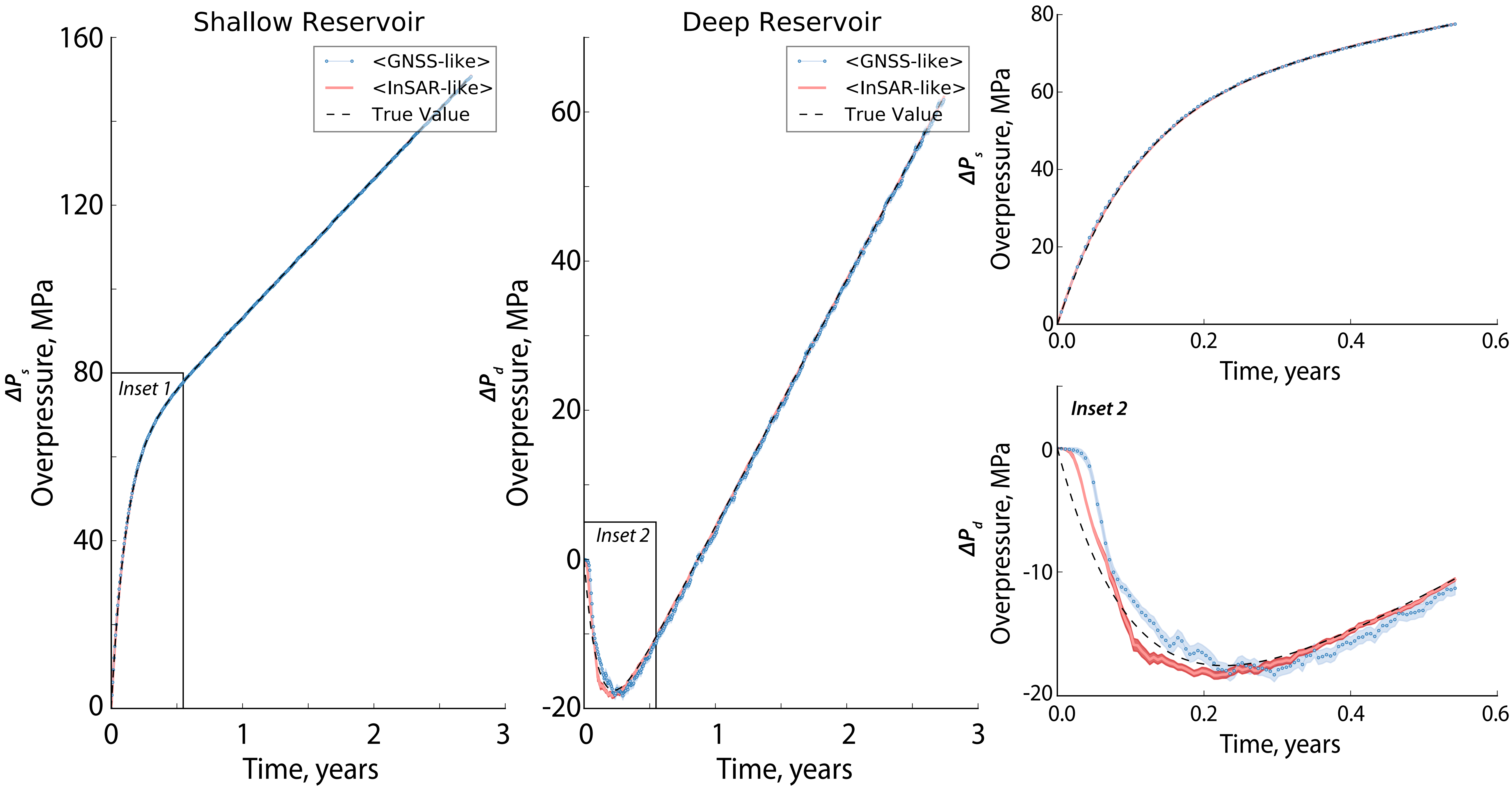
InSAR dataset:

The assimilation interval, $\Delta t = 2 \text{ days}$

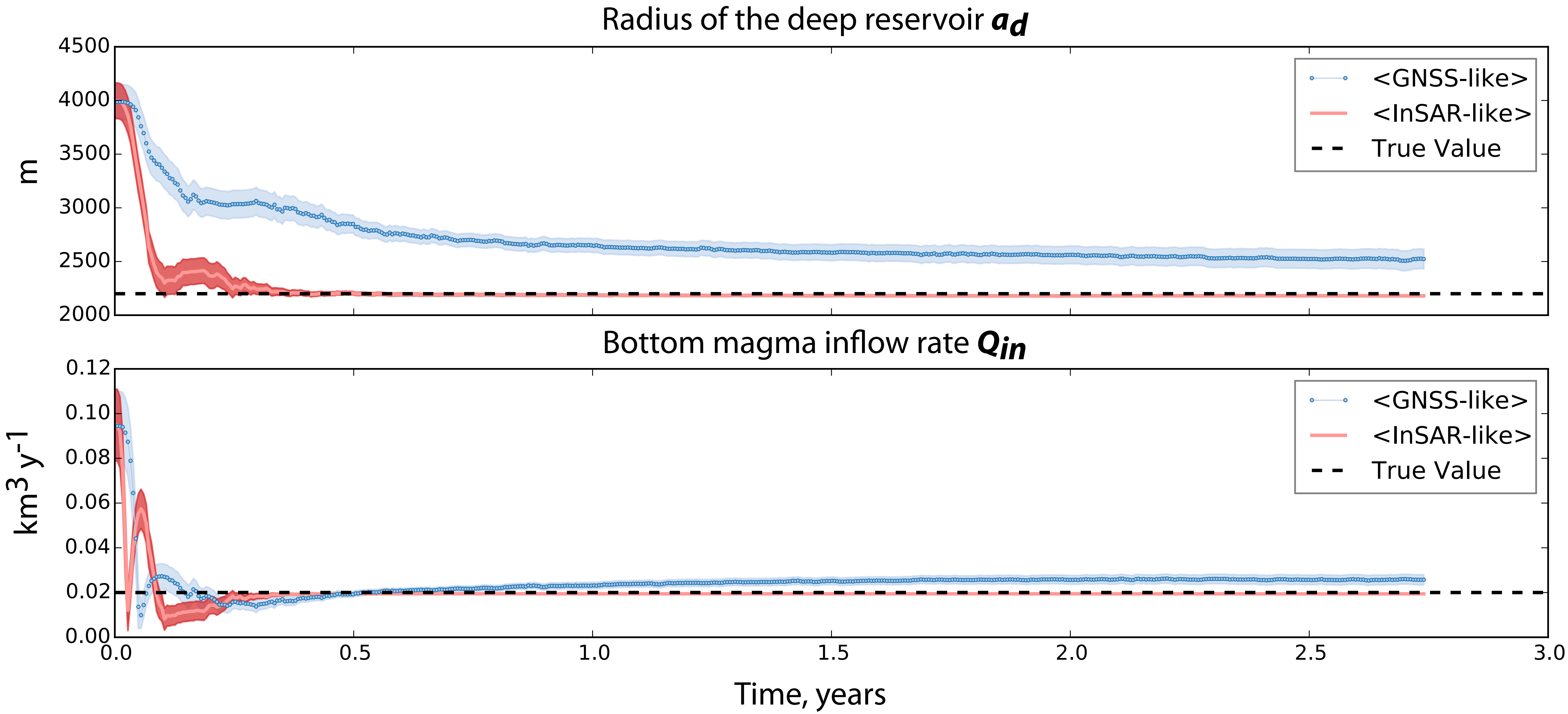
The frequency of available observation is also 2 days.

242 observations are used for the synthetic cases.

- ☑ 11x11 radial and 11x11 vertical

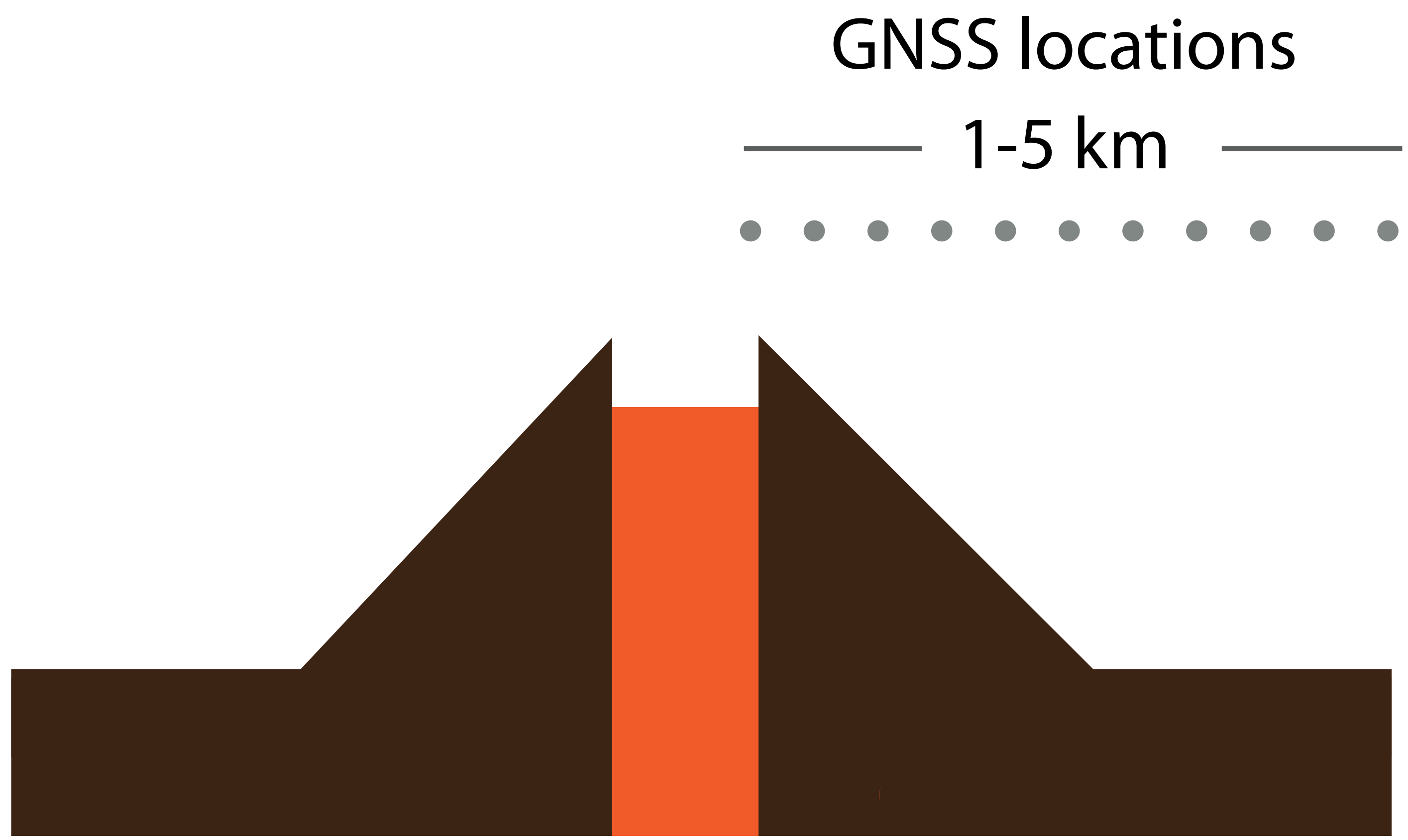


Shallow Reservoir
Deep Reservoir
<GNSS-like>
<InSAR-like>
True Value
<GNSS-like>
<InSAR-like>
True Value
Inset 1
Inset 2
Inset 2
ΔP_s Overpressure, MPa
ΔP_d Overpressure, MPa
ΔP_s Overpressure, MPa
ΔP_d Overpressure, MPa
Time, years
Time, years
Time, years
Time, years
160
120
80
40
0
60
40
20
0
-20
80
60
40
20
0
0
-10
-20
0
1
2
3
0
1
2
3
0.0
0.2
0.4
0.6
0.0
0.2
0.4
0.6



Radius of the deep reservoir a_d
4500
4000
3500
3000
2500
2000
m
<GNSS-like>
<InSAR-like>
True Value
Bottom magma inflow rate Q_{in}
0.12
0.10
0.08
0.06
0.04
0.02
0.00
km^3 y^{-1}
<GNSS-like>
<InSAR-like>
True Value
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time, years

GNSS VS. INSAR:
HOW **TEMPORAL** RESOLUTION AFFECTS THE ASSIMILATION



GNSS dataset:

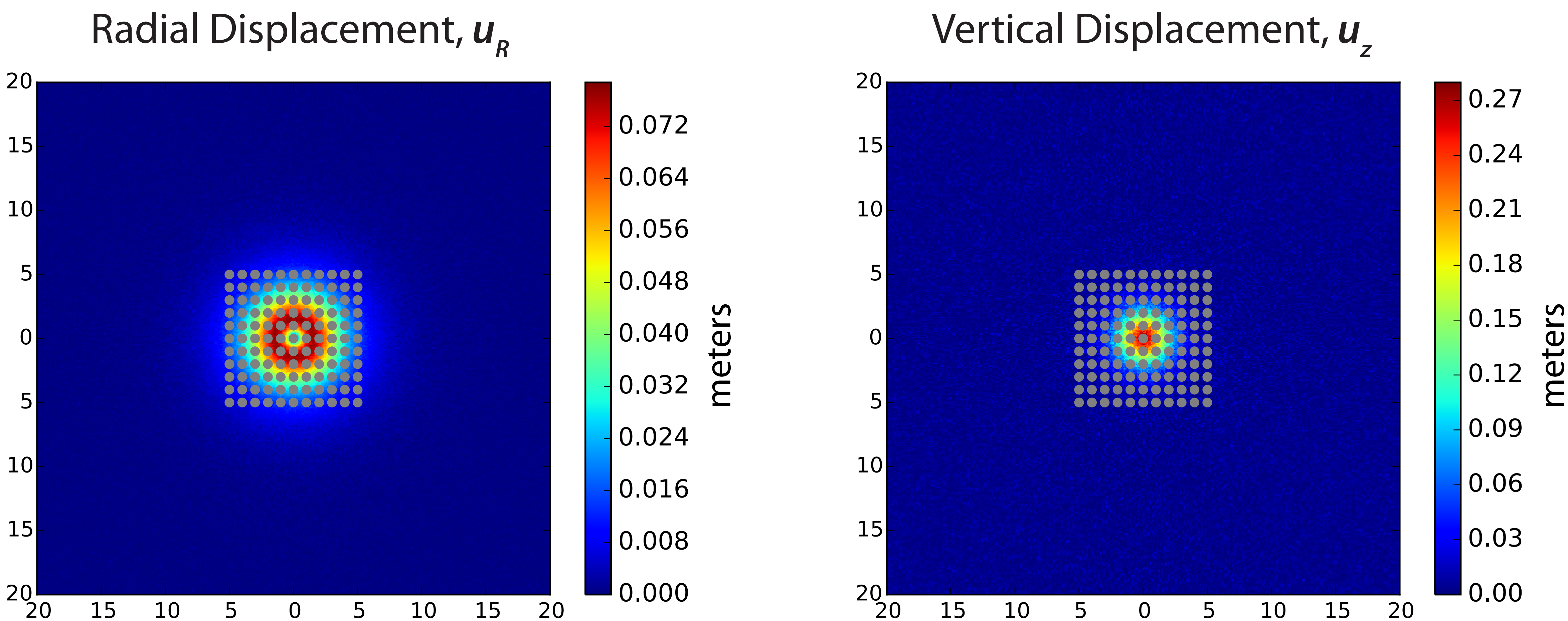
The assimilation interval, $\Delta t = 2 \text{ days}$

The frequency of available observation is also **2 days**.

10 observations are used for the synthetic cases.

- 5 radial and 5 vertical

GNSS VS. INSAR:
HOW **TEMPORAL** RESOLUTION AFFECTS THE ASSIMILATION



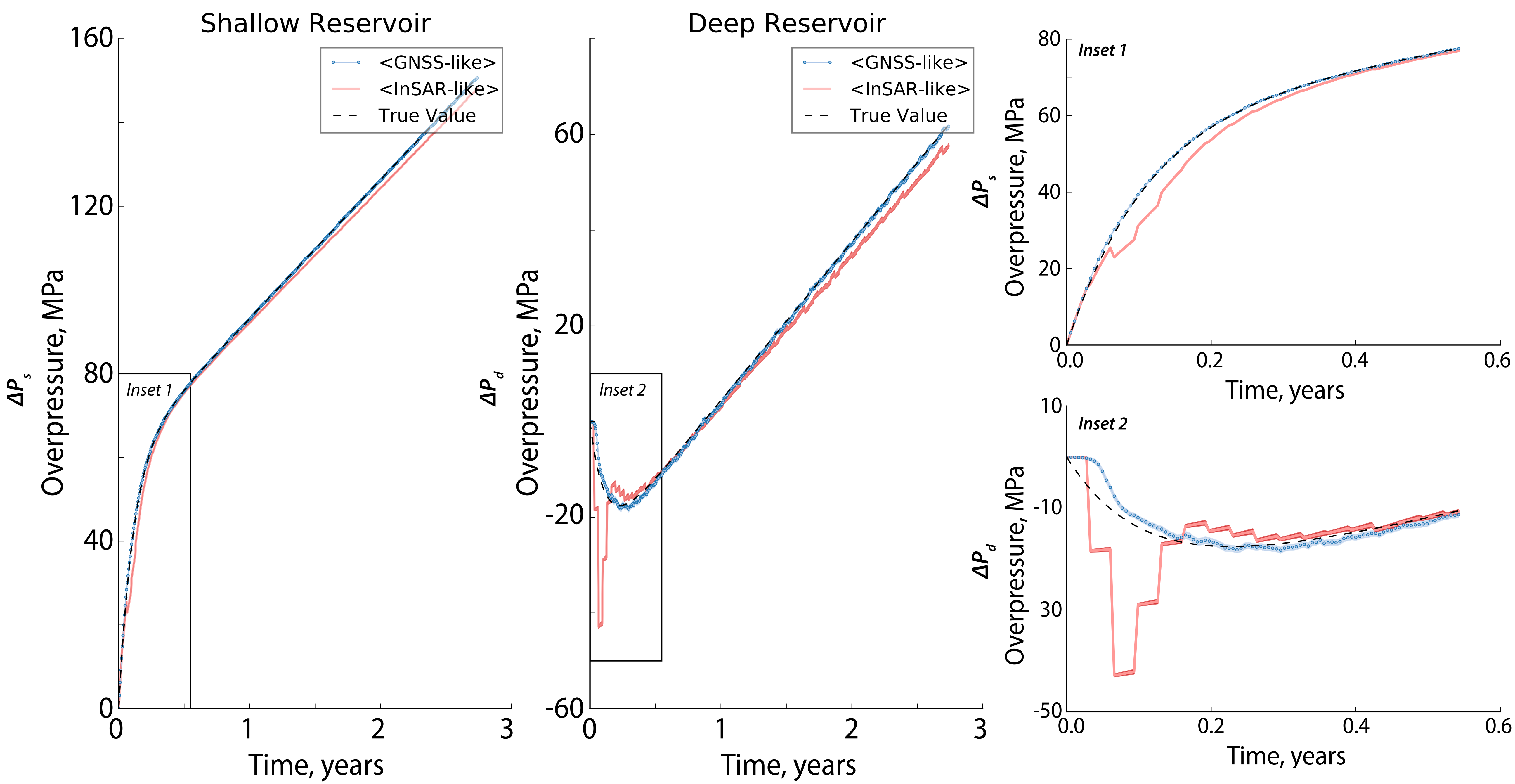
InSAR dataset:

The assimilation interval, $\Delta t = 2$ days

The frequency of available observation is **every 12 days.**

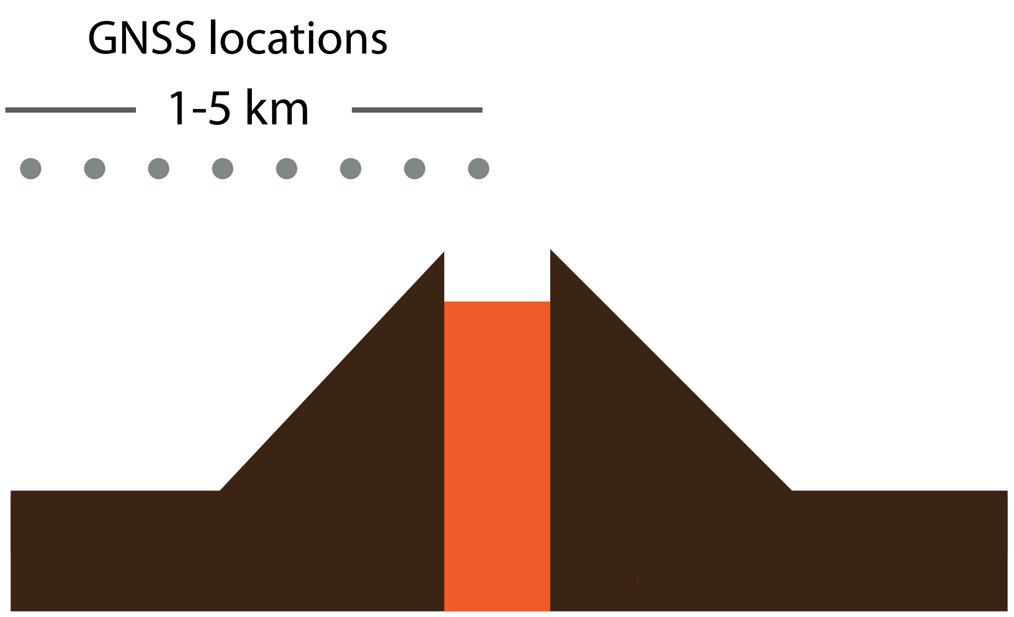
242 observations are used for the synthetic cases.

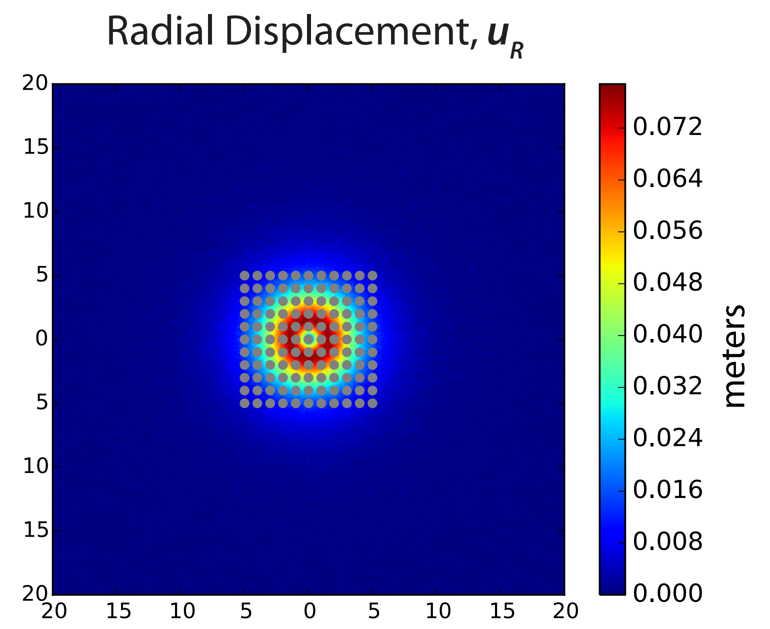
- ☑ 11x11 radial and 11x11 vertical

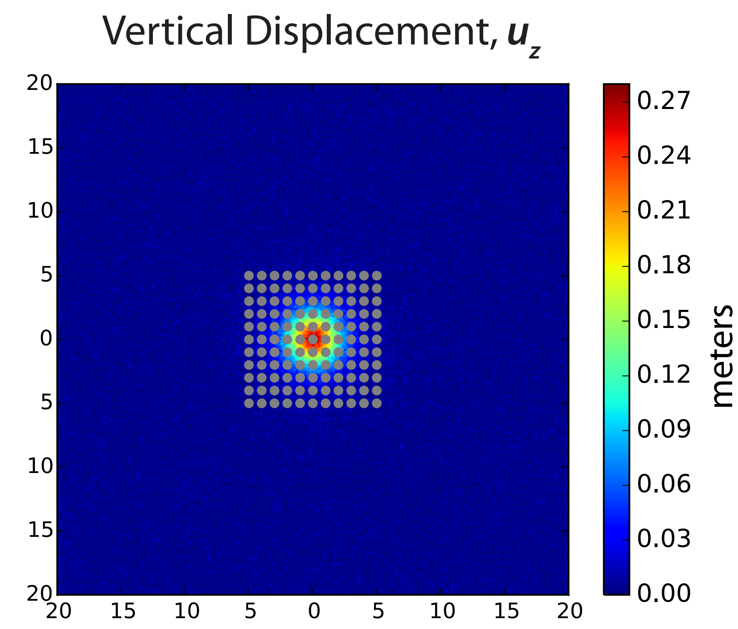


Shallow Reservoir
Deep Reservoir
<GNSS-like>
<InSAR-like>
True Value
Inset 1
Inset 2
ΔP_s
ΔP_d
Overpressure, MPa
Time, years

JOINT ASSIMILATION
GNSS AND INSAR







GNSS dataset:

The assimilation interval, $\Delta t = 2$ days

The frequency of available observation **every 2 days.**

10 observations are used for the synthetic cases.

- ☑ 5 radial and 5 vertical

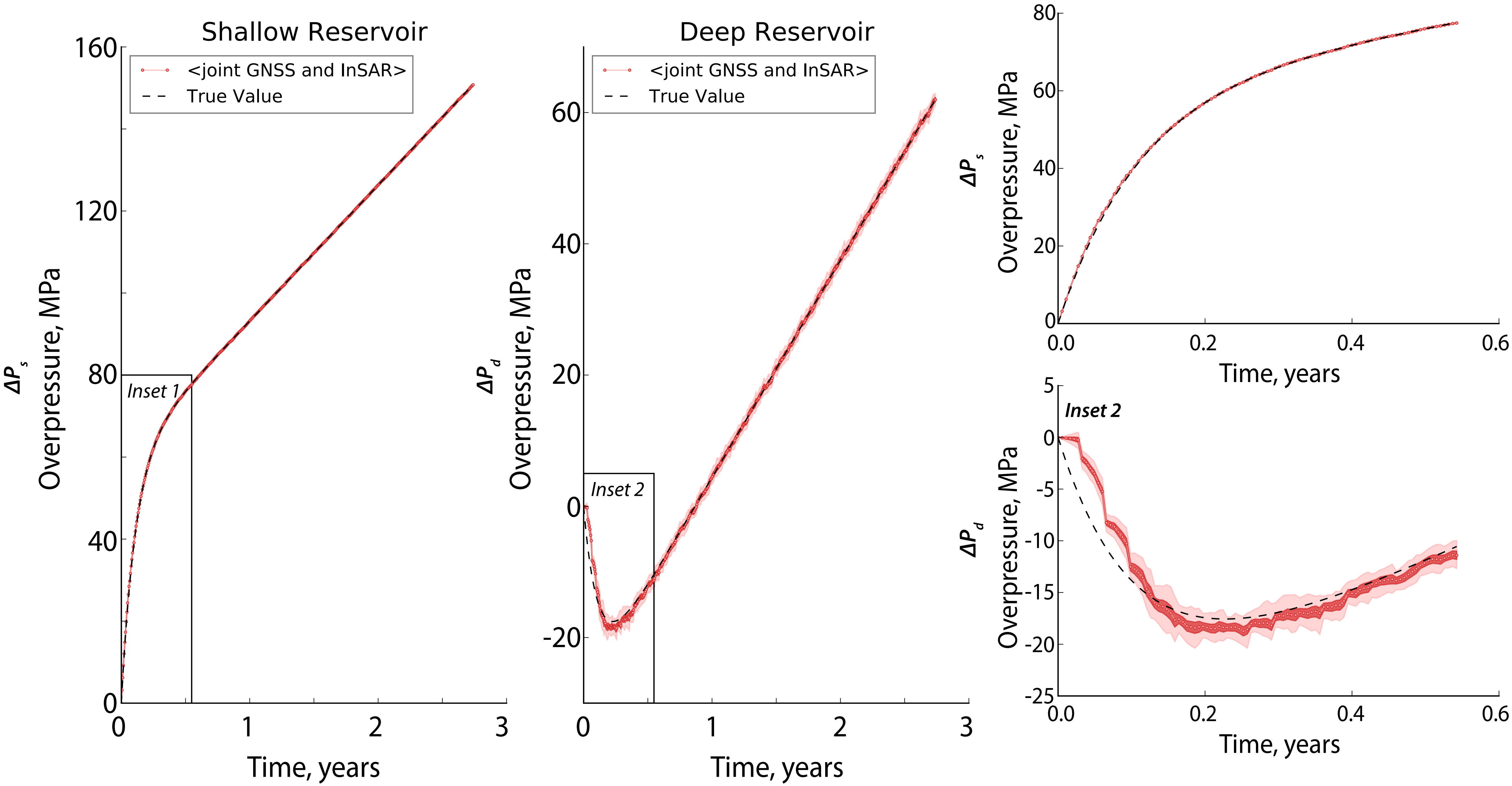
InSAR dataset:

The assimilation interval, $\Delta t = 12$ days

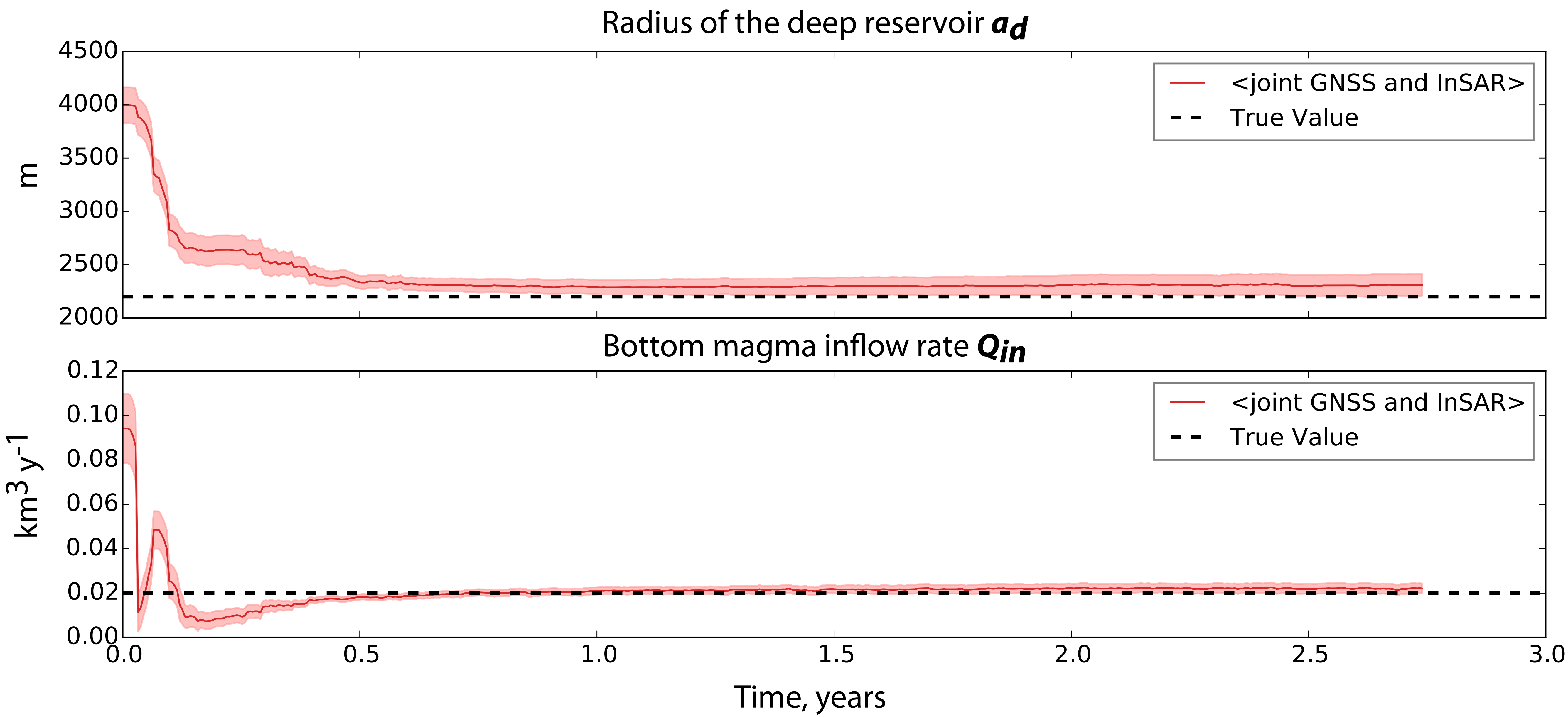
The frequency of available observation is **every 12 days.**

242 observations are used for the synthetic cases.

- ☑ 11x11 radial and 11x11 vertical

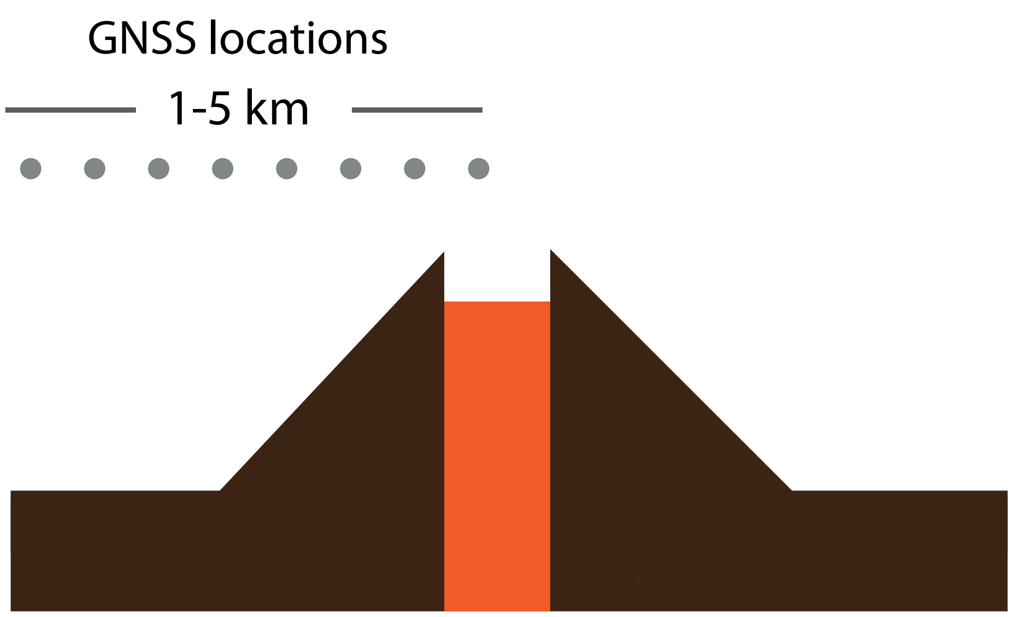


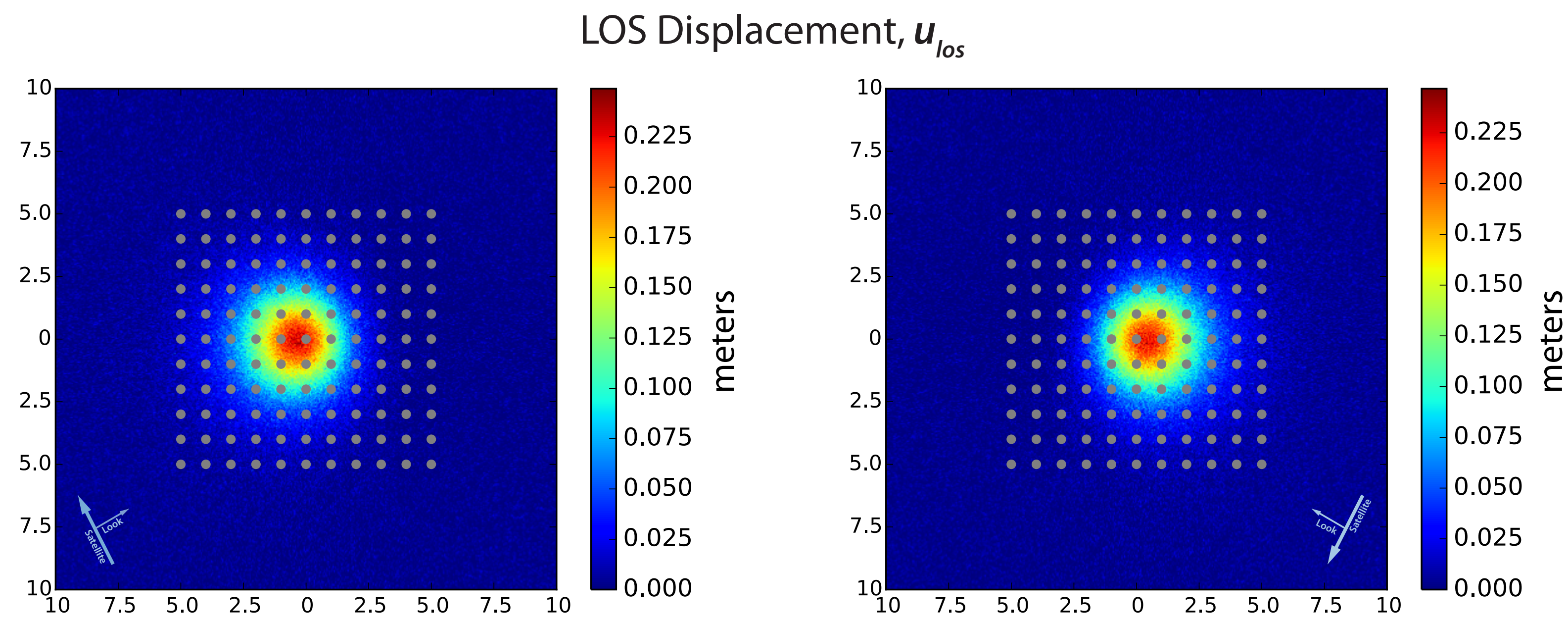
Shallow Reservoir
Deep Reservoir
<joint GNSS and InSAR>
True Value
ΔP_s Overpressure, MPa
ΔP_d Overpressure, MPa
Time, years
Inset 1
Inset 2



Radius of the deep reservoir a_d
4500
4000
3500
3000
2500
2000
m
<joint GNSS and InSAR>
True Value
Bottom magma inflow rate Q_{in}
0.12
0.10
0.08
0.06
0.04
0.02
0.00
km^3 y^{-1}
<joint GNSS and InSAR>
True Value
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time, years

JOINT ASSIMILATION
GNSS AND INSAR-LOS





GNSS dataset:

The assimilation interval, $\Delta t = 2$ days

The frequency of available observation **every 2 days.**

10 observations are used for the synthetic cases.

- 5 radial and 5 vertical

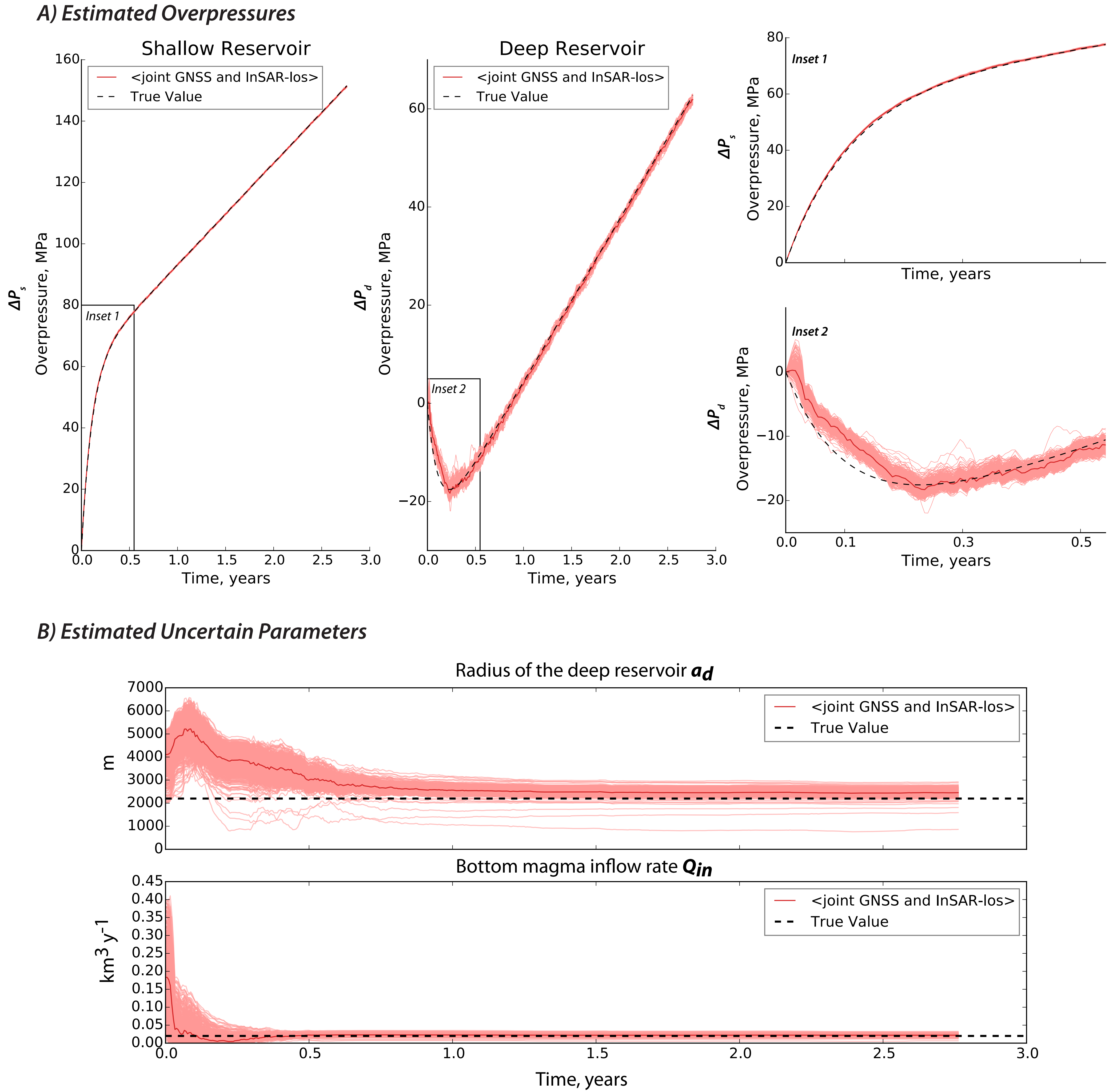
InSAR dataset:

The assimilation interval, $\Delta t = 12$ days

The frequency of available observation is **every 12 days.**

121 observations are used for the synthetic cases.

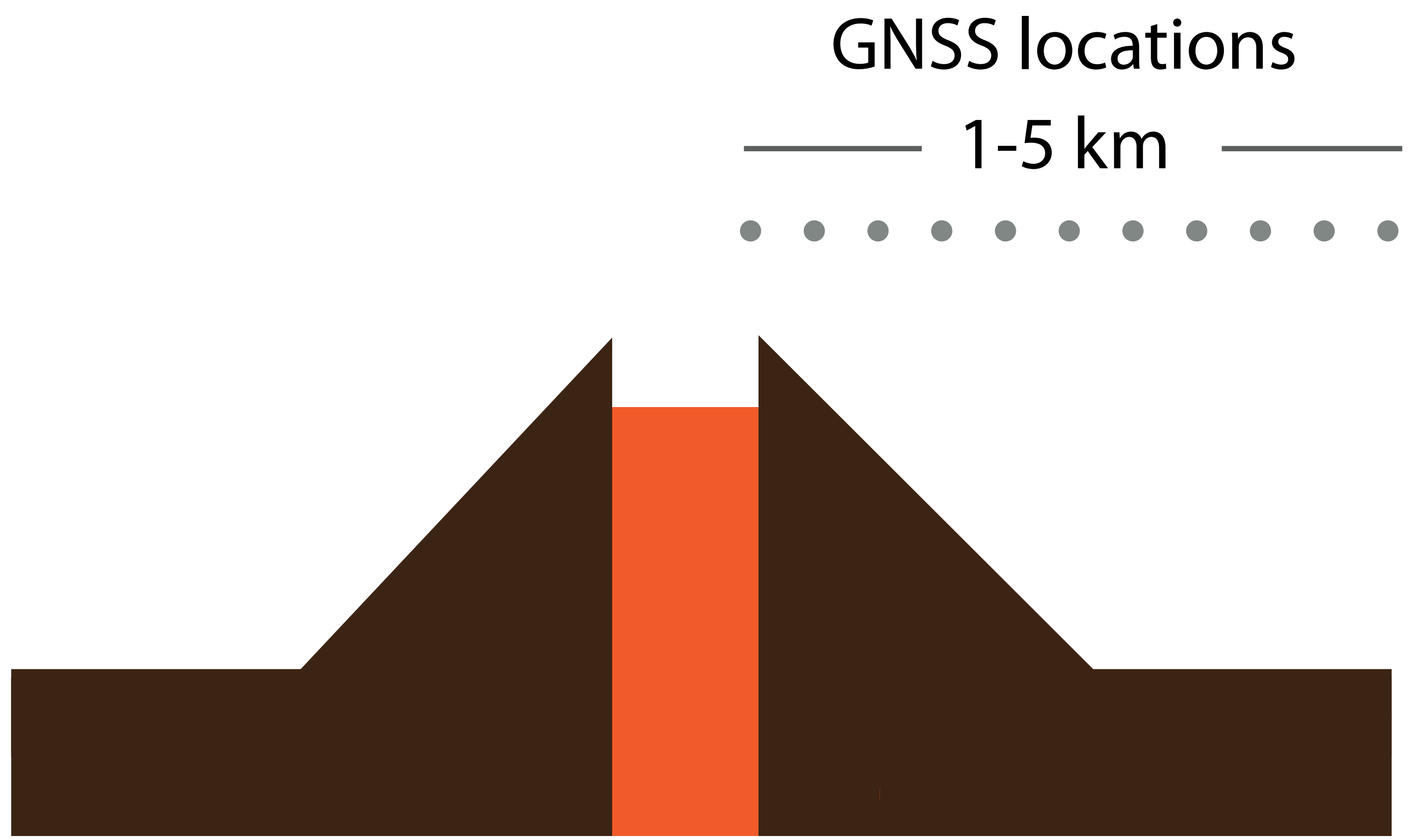
- 11x11 LOS data in either ascending or descending pass



A) Estimated Overpressures
Shallow Reservoir
Deep Reservoir
<joint GNSS and InSAR-los>
True Value
ΔP_s Overpressure, MPa
ΔP_d Overpressure, MPa
Inset 1
Inset 2
Time, years
B) Estimated Uncertain Parameters
Radius of the deep reservoir a_d
m
Bottom magma inflow rate Q_{in}
km^3 y^{-1}
Time, years

ENKF VS. BAYESIAN INVERSION

ENKF VS. BAYESIAN INVERSION

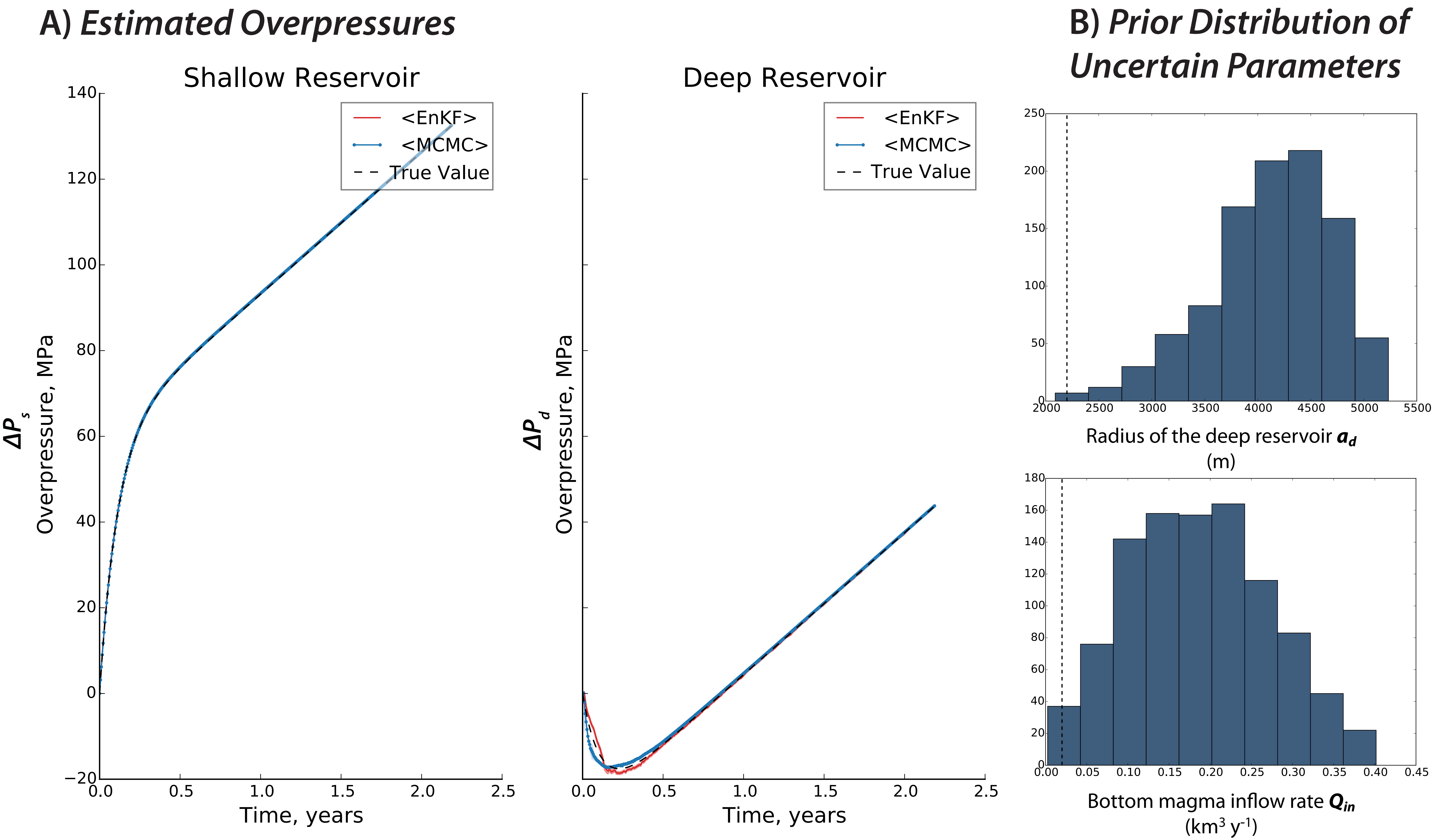


The assimilation interval, $\Delta t = 2 \text{ days}$

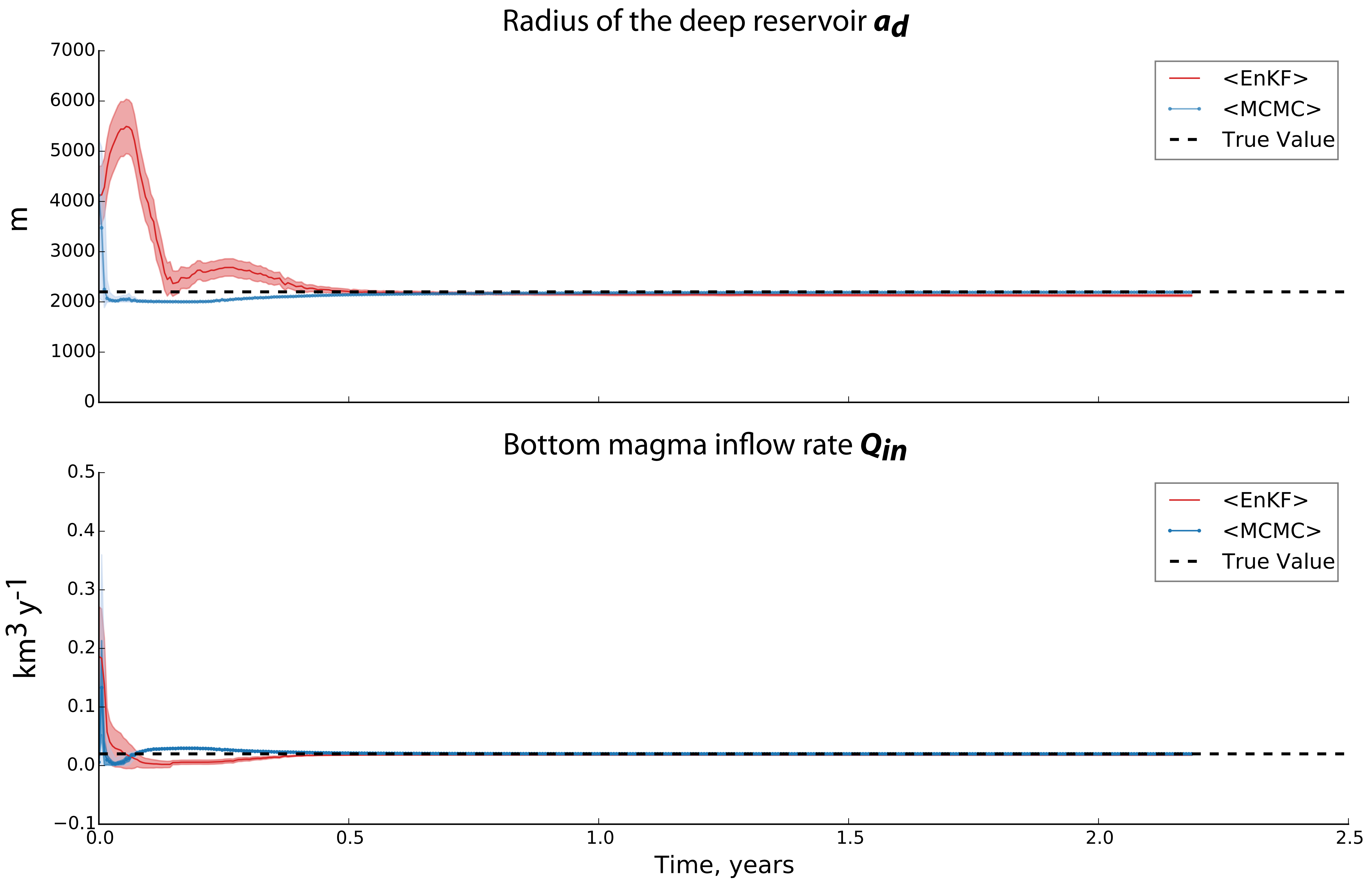
The frequency of available observation is also 2 days.

80 observations are used for the synthetic cases.

- ☑ 40 radial and 40 vertical



A) Estimated Overpressures
Shallow Reservoir
Deep Reservoir
<EnKF>
<MCMC>
True Value
ΔP_s Overpressure, MPa
ΔP_d Overpressure, MPa
Time, years
B) Prior Distribution of Uncertain Parameters
Radius of the deep reservoir a_d (m)
Bottom magma inflow rate Q_in (km³ y⁻¹)

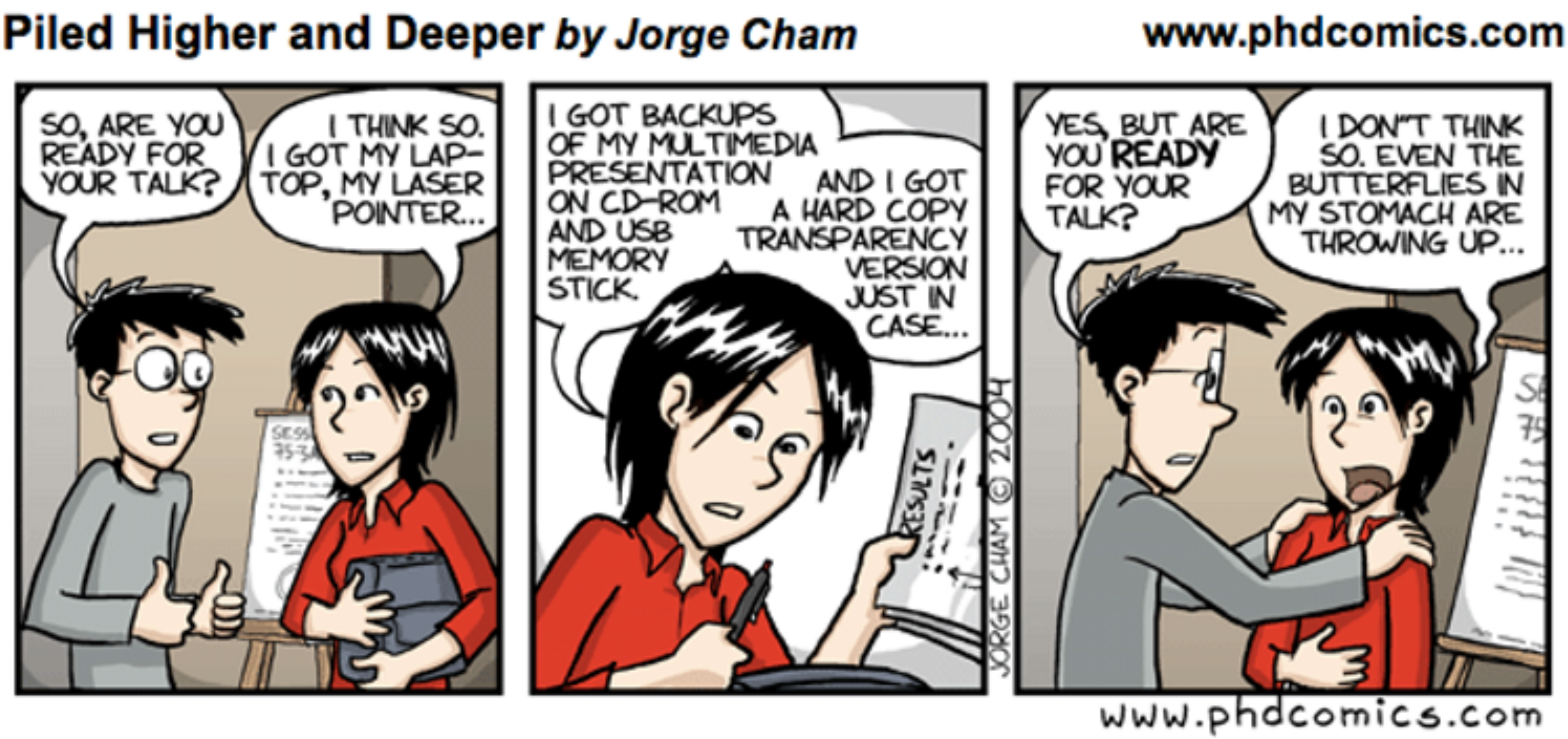


Radius of the deep reservoir a_d
m
7000
6000
5000
4000
3000
2000
1000
0
<EnKF>
<MCMC>
True Value
Bottom magma inflow rate Q_{in}
$km^3\ y^{-1}$
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.5
1.0
1.5
2.0
2.5
Time, years
<EnKF>
<MCMC>
True Value

KEY POINTS

- **Efficient model-data fusion technique to forecast effusive eruptions**
  - Overpressures can be estimated if:
    - Prior information about uncertain parameters are well constrained
    - If uncertain parameters are also estimated
- **GNSS vs. InSAR**
  - GNSS can recover the true evolution of the overpressures because of its good temporal resolution
  - InSAR can better constrain uncertain parameters because of its high spatial resolution
  - Joint assimilation of these datasets are successfully presented for the first time
- **Data assimilation (EnKF) vs. Bayesian-based inversion (MCMC)**
  - Similar capabilities when estimating uncertain parameters
  - EnKF can be used to forecast in near-real time
  - EnKF may be able to track time-dependent uncertain parameters
- **Framework is simple yet offers a great potential towards a more deterministic eruption forecasting and better understanding of the magma plumbing system**

Merci !



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