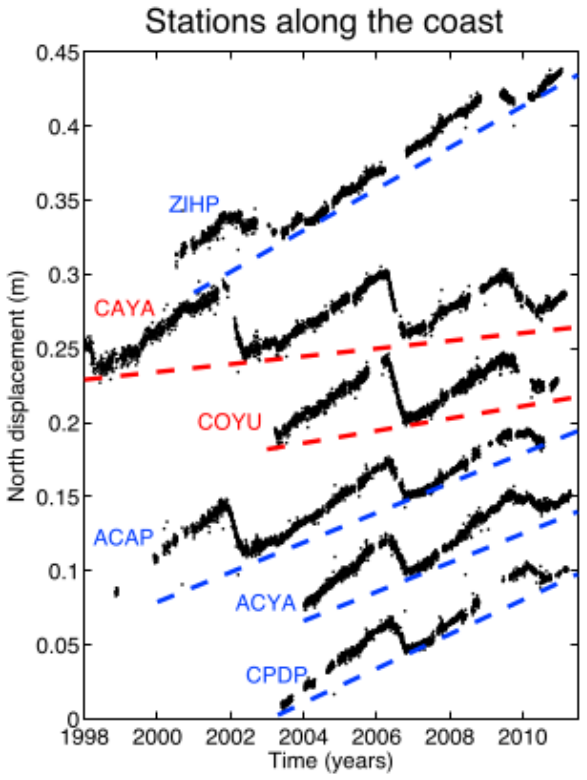
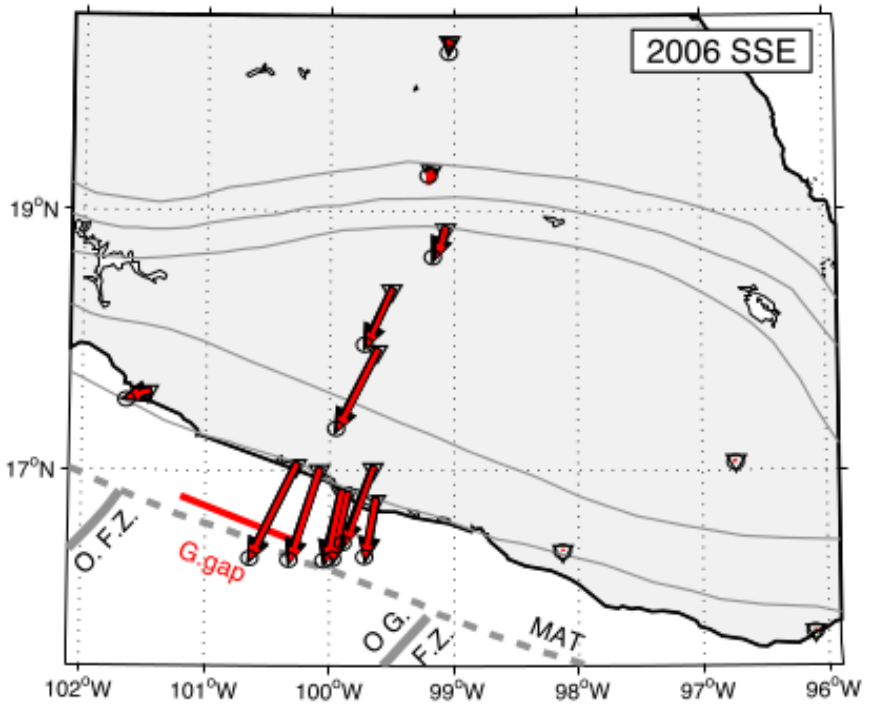
Data adaptive analysis of geodetic time series

Damian Walwer, Eric Calais, Michael Ghil
Département de Géosciences de l'ENS

MDIS - 18 Octobre 2017

Deformation time series



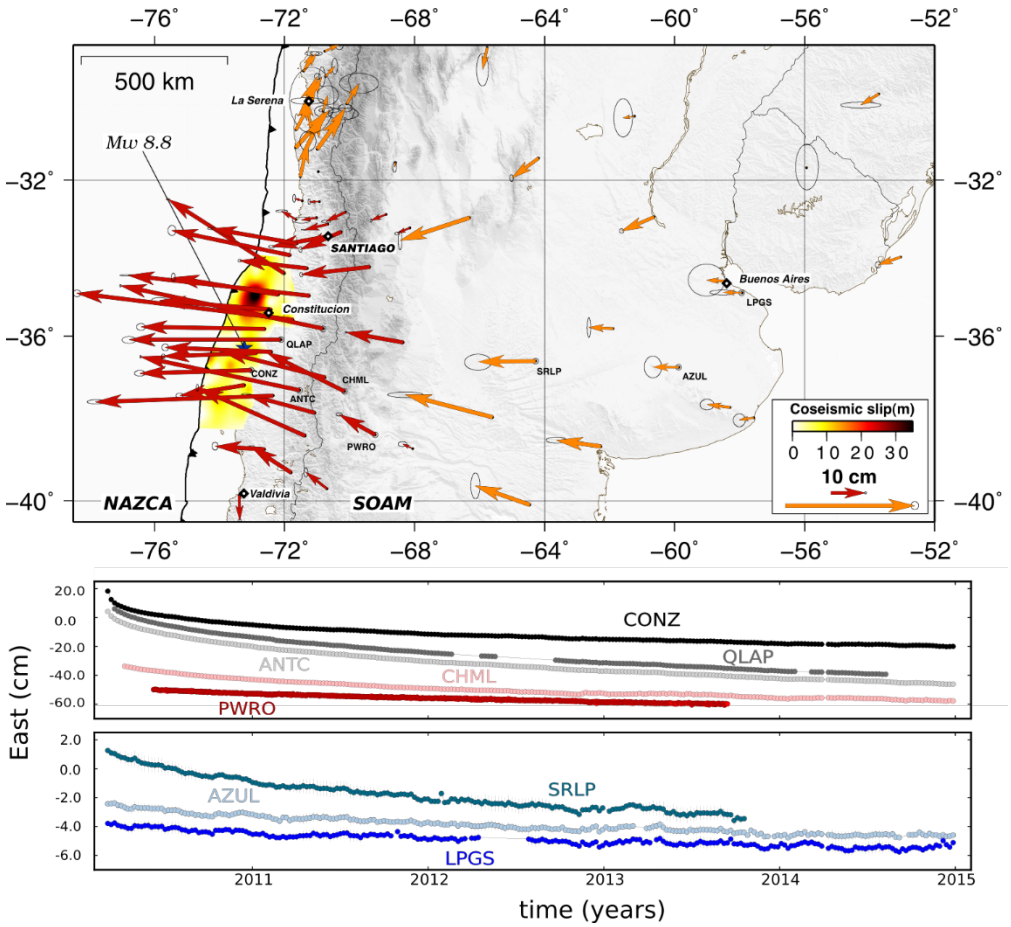


Quasi periodic occurrences of slow slips at the Guerrero subduction zone

Usefull to study faults systems ***dynamics***

Radiguet et al., 2012

Deformation time series

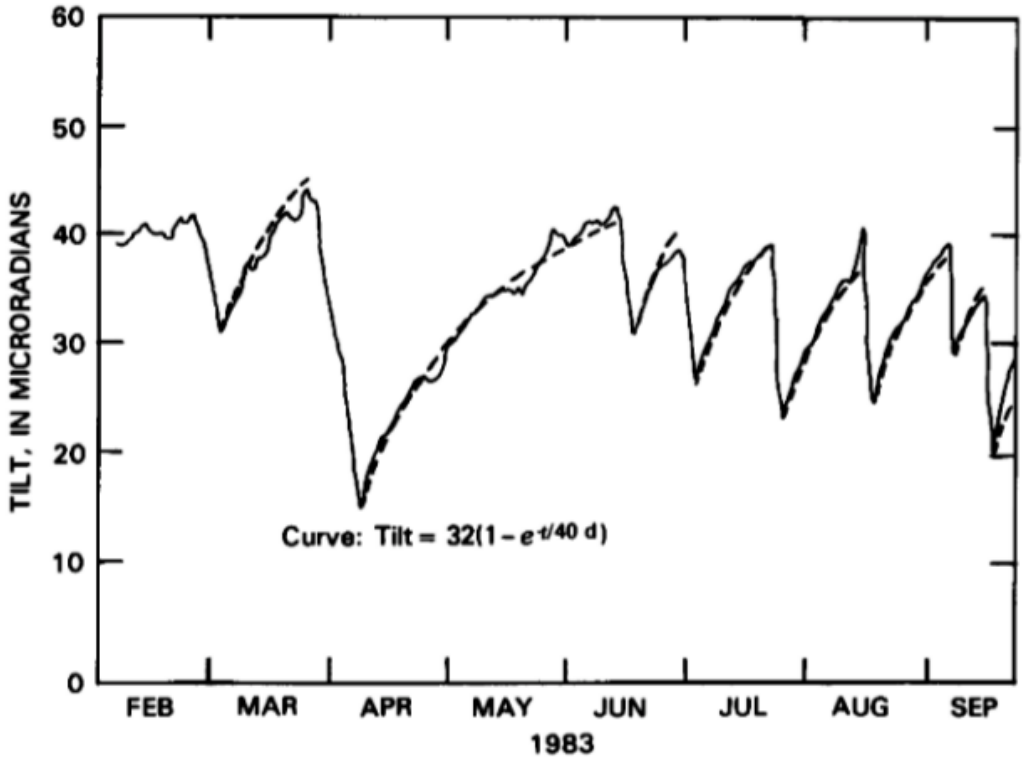


Postseismic deformation recorded after the Maule earthquake (Mw 8.8), Chile, 2010

Usefull to constrain the ***dynamics*** of viscous relaxation of the mantle after large earthquake

Klein et al. 2016

Deformation time series



Inflation and deflation of Kilauea summit

Allow to study the ***dynamics*** of eruptive cycles

Dvorak and Okamura, 1987

Deformation time series

Give information about the **qualitative dynamics** of geological systems

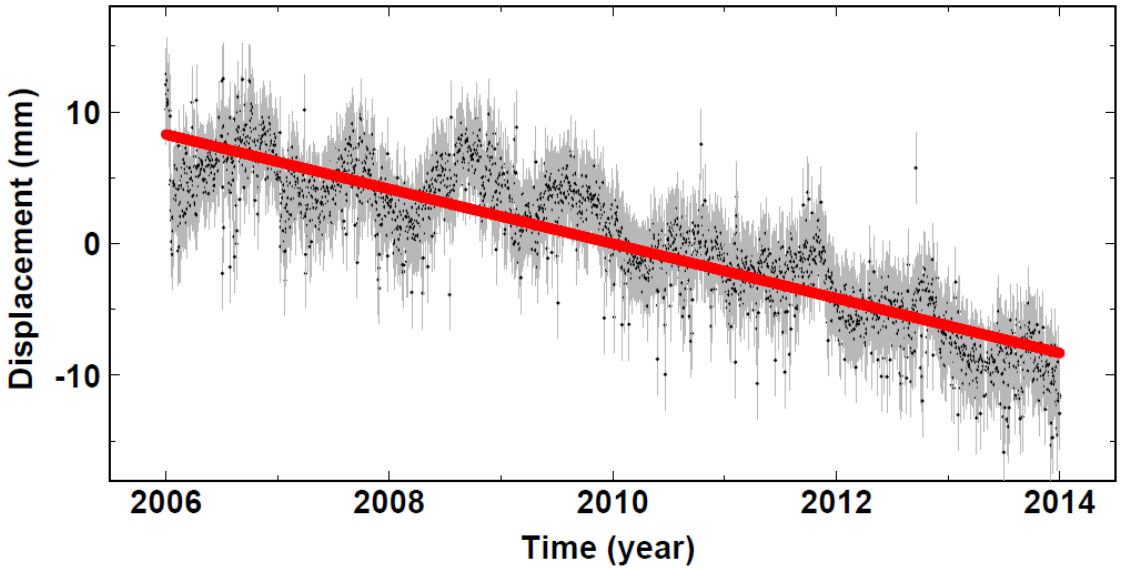
$$\frac{d\mathbf{y}}{dt} = \mathbf{F}(\mathbf{y})$$

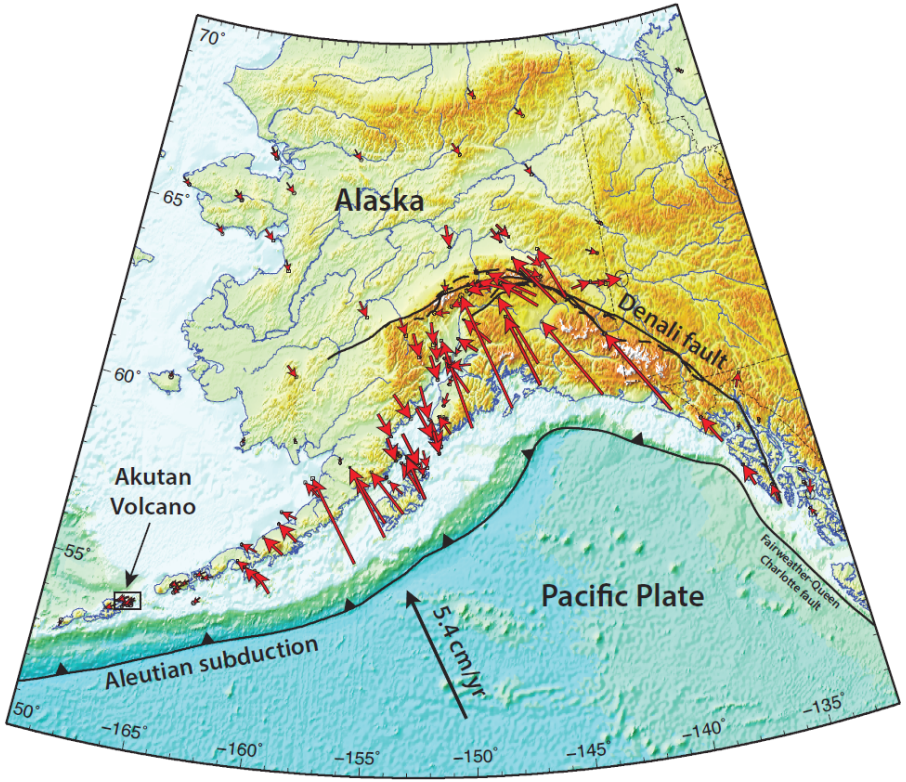
$\mathbf{y} = (y_1, y_2, \ldots)$ variables to specify the state of a system

Not so easy sometimes to access **qualitative dynamics !**

see e.g. Broomhead and King, 1986 ; Vautard and Ghil, 1989

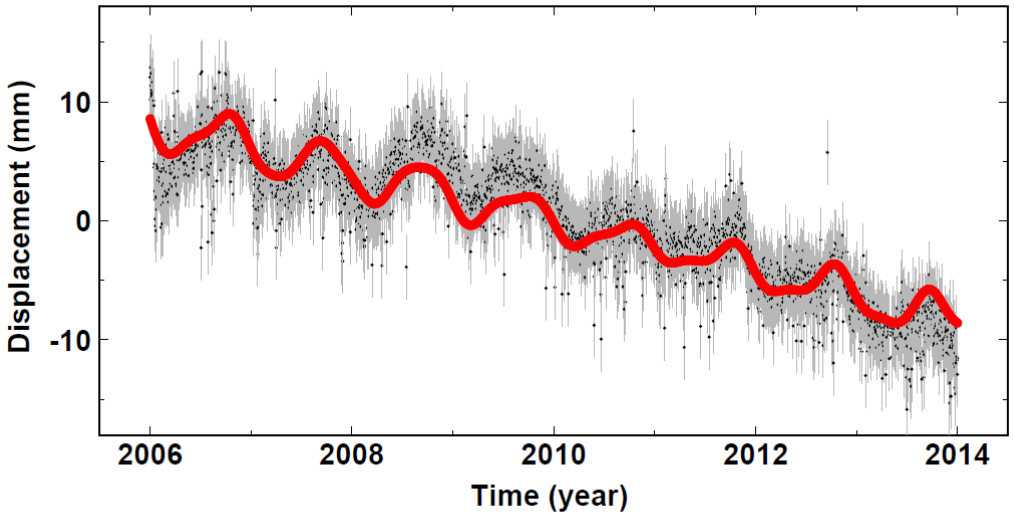
GPS time series

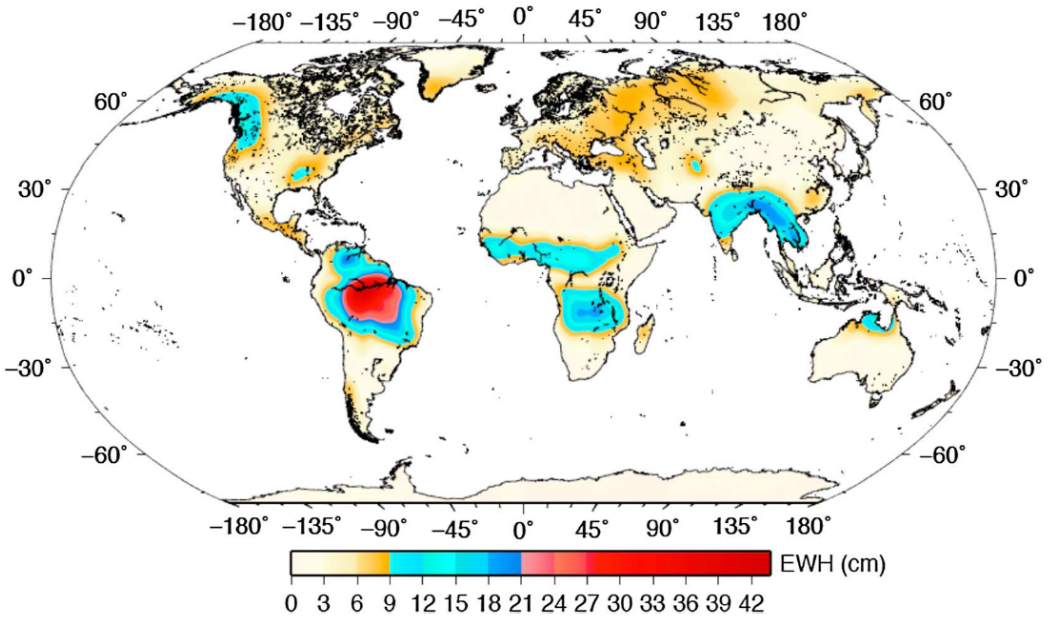




First order : linear displacement associated with rigid mouvements of plates or elastic strain accumulation near active faults

GPS time series

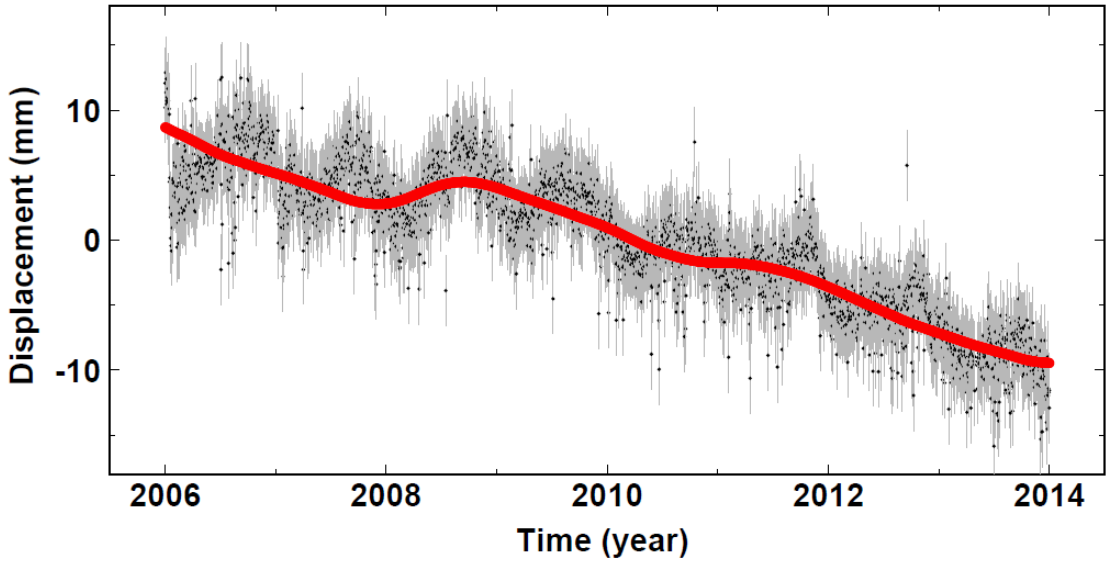




Fu et al. 2013

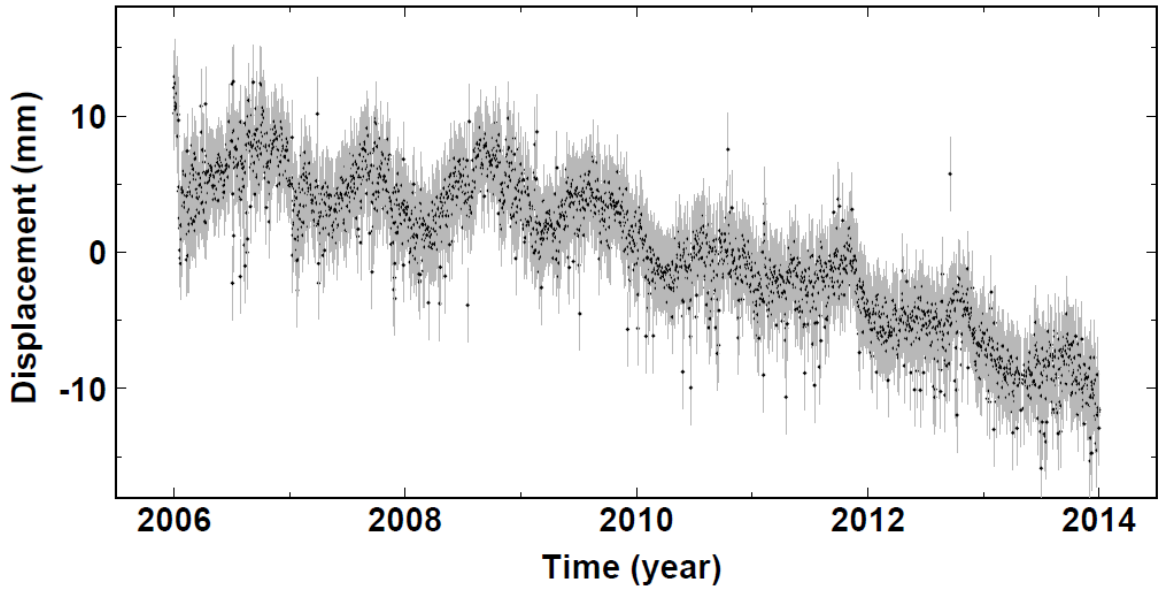
- Seasonal oscillations (annual and semiannual)
- Amplitude : millimiter / centimeter
- Cause : Elastic loads triger by atmospheric, oceanic and hydrologic mass mouvement

GPS time series



- Transient mouvements
- Amplitude : variable
- Periode : variable
- Cause : volcanic processes, tectonic (slow earthquakes), hydrology ....

GPS time series

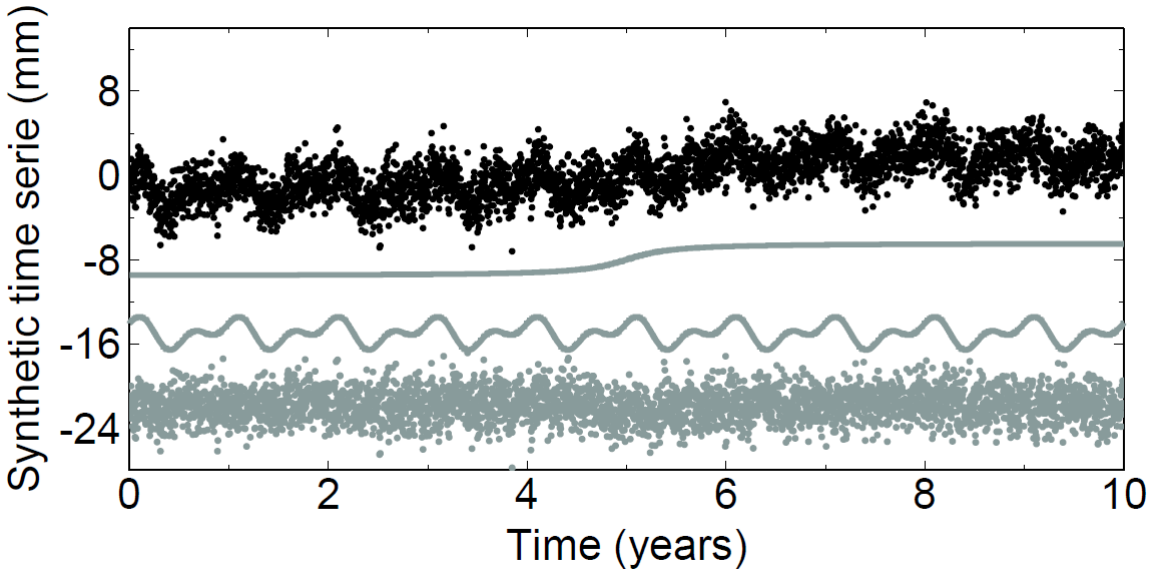


Challenge : identify these structures, separate them from the noise as well as from other signals

How to do this?

The M-SSA (Multichannel Singular Spectrum Analysis)

Exploit simultaneously the spatial and temporal correlations of geophysical fields



$$\tilde{\mathbf{X}}_l = \begin{pmatrix} X_l(1) & X_l(2) & \cdots & X_l(M) \\ X_l(2) & X_l(3) & \cdots & X_l(M+1) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ X_l(N') & X_l(N'+1) & \cdots & X_l(N) \end{pmatrix}$$

The M-SSA

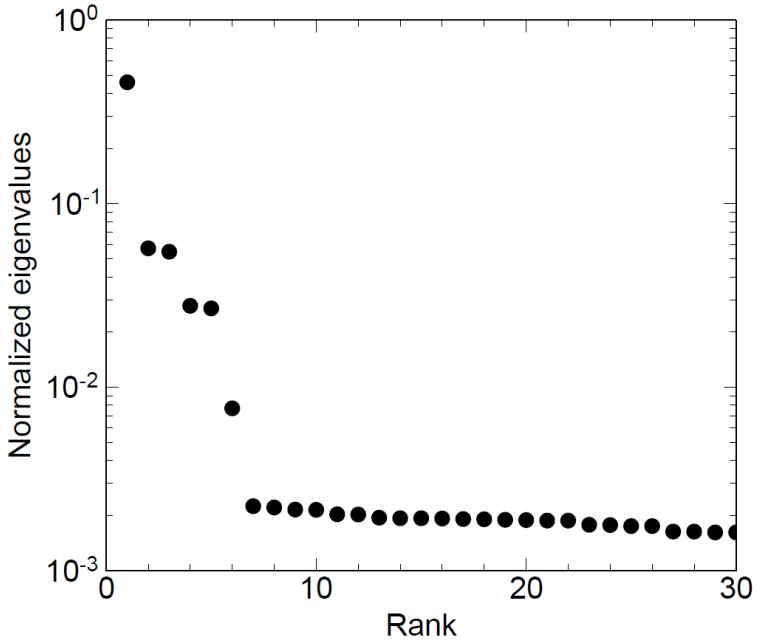
$$\tilde{\mathbf{T}} = \begin{pmatrix} \mathbf{T}_{1,1} & \mathbf{T}_{1,2} & \cdot & \cdot & \cdot & \mathbf{T}_{1,L} \\ \mathbf{T}_{2,1} & \mathbf{T}_{2,2} & \cdot & \cdot & \cdot & \mathbf{T}_{2,L} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \mathbf{T}_{l,l'} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathbf{T}_{L,1} & \mathbf{T}_{L,2} & \cdot & \cdot & \cdot & \mathbf{T}_{L,L} \end{pmatrix}$$

The M-SSA

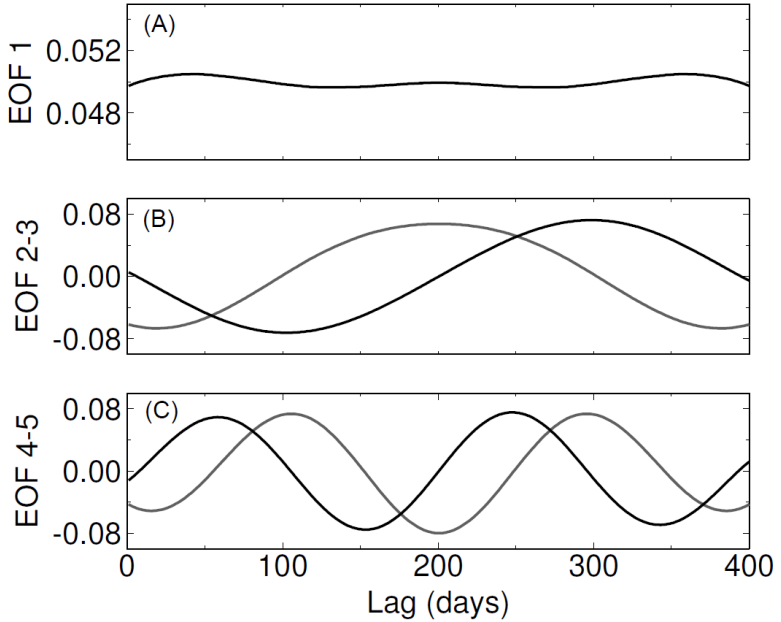
The resolution of the eigen value problem

$$\mathbf{\Lambda} = \mathbf{E}'\tilde{\mathbf{T}}\mathbf{E},$$

gives



Eigen values

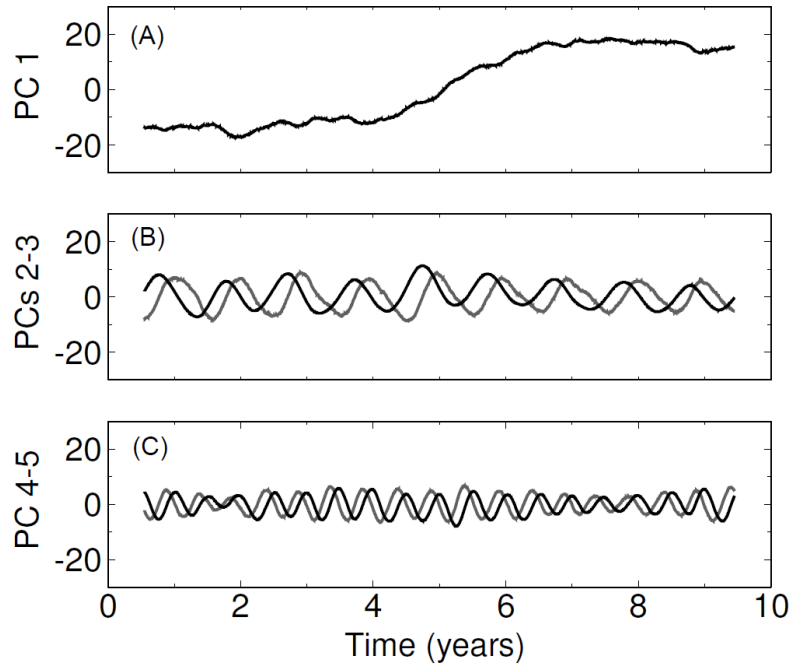


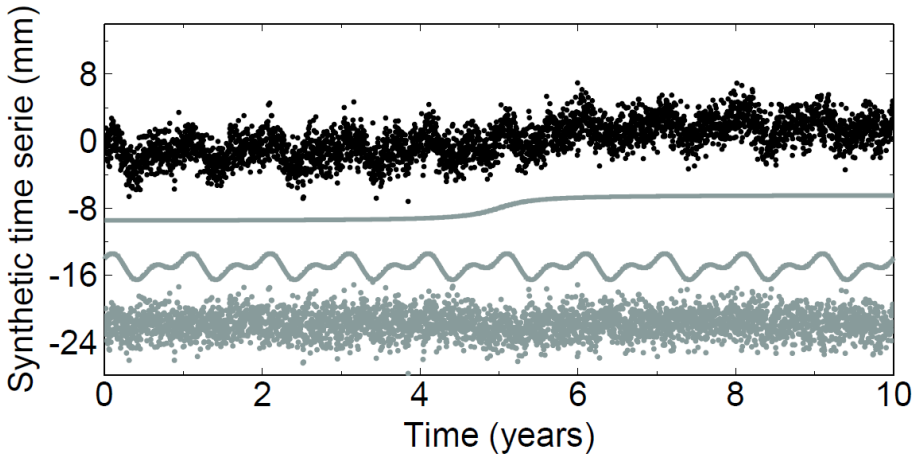
Eigen vectors : Empirical orthogonal functions (EOF)

The M-SSA

Projection of the time series on the EOF gives the Principal components (PC)

$$\mathbf{A} = \mathbf{XE};$$

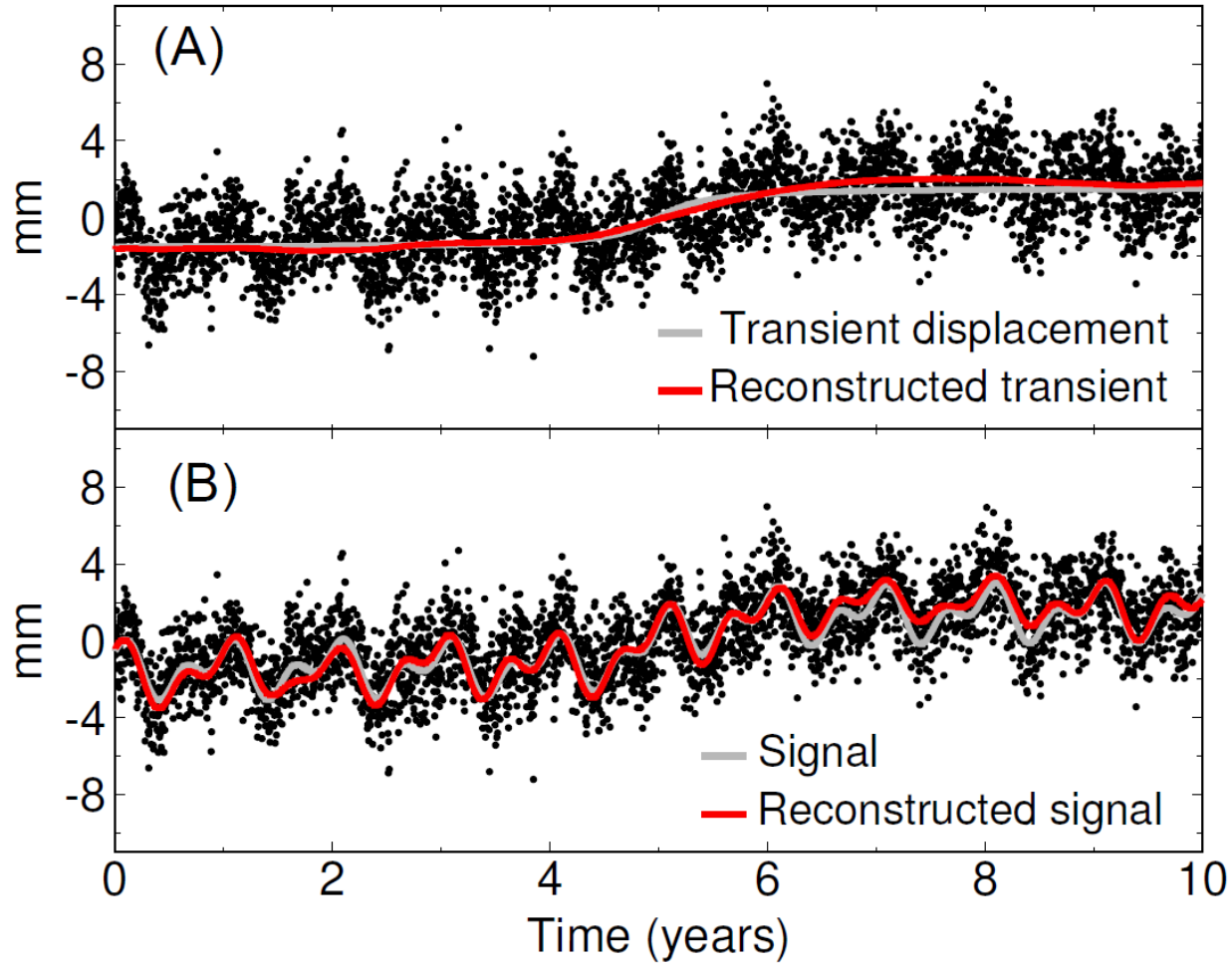




The M-SSA

$$\mathbf{R} = \mathbf{A}\,\mathbf{K}\,\mathbf{E}'.$$

$$\mathcal{K} \subset \{1 \ldots LM\}$$

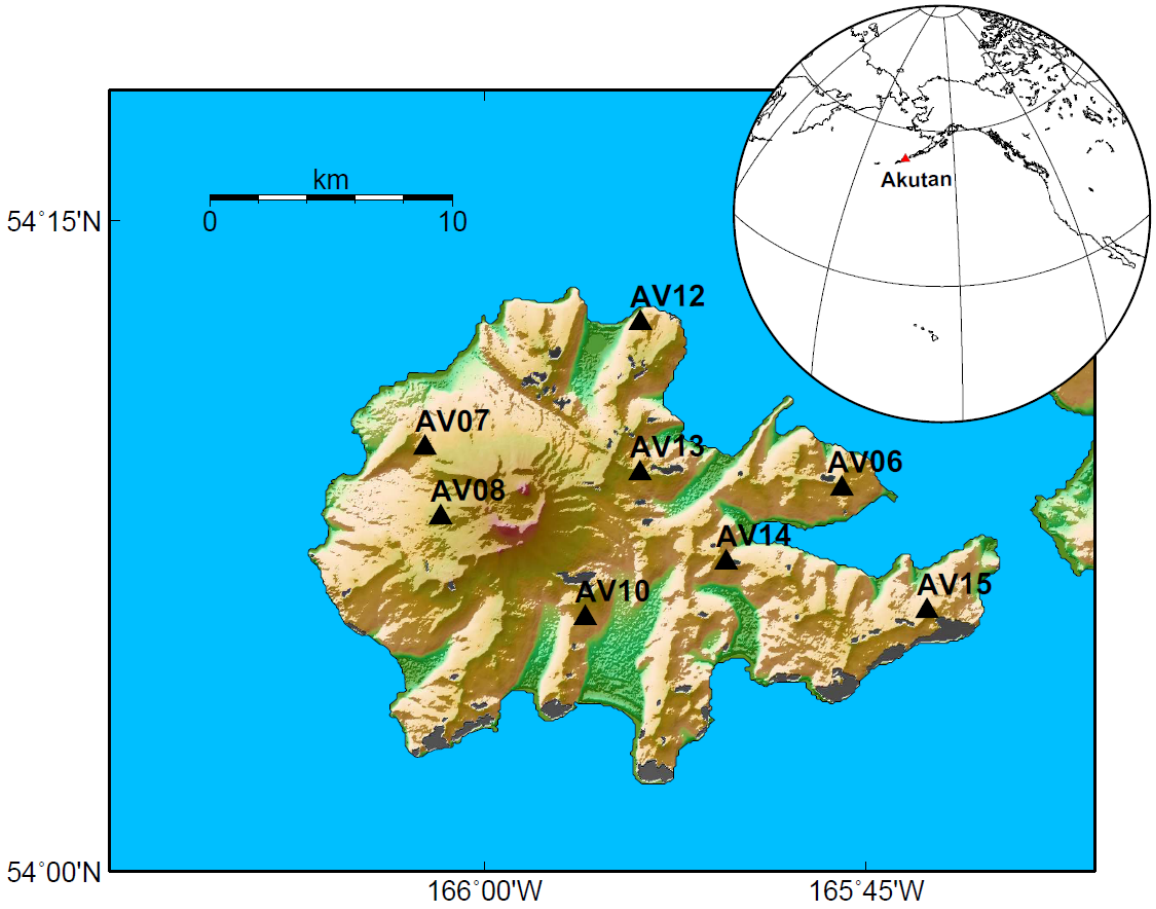


The M-SSA

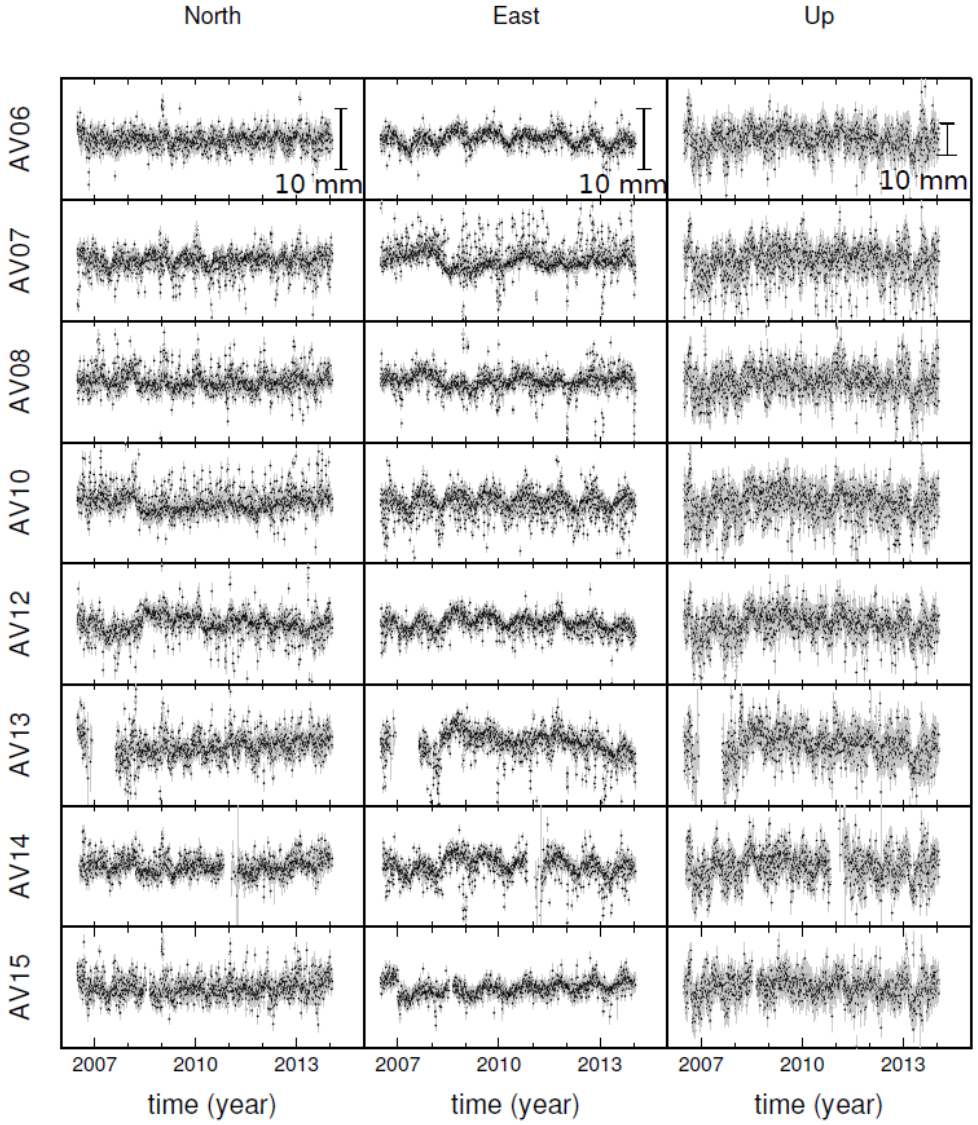
- The M-SSA exploit simultaneously the spatial and temporal correlations of GPS time series

- Define Empirical eigen vectors = Data-adaptive = No use of a priori hypothesis

Akutan volcano

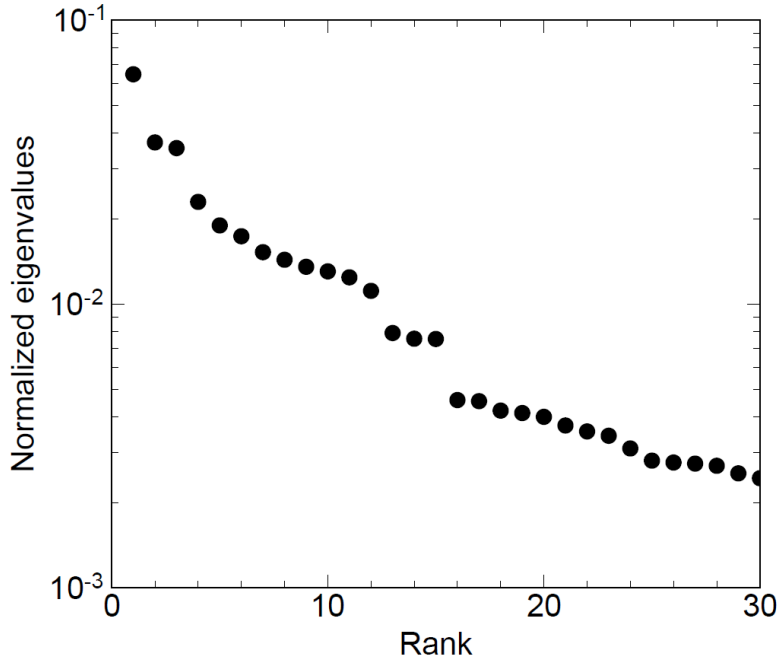


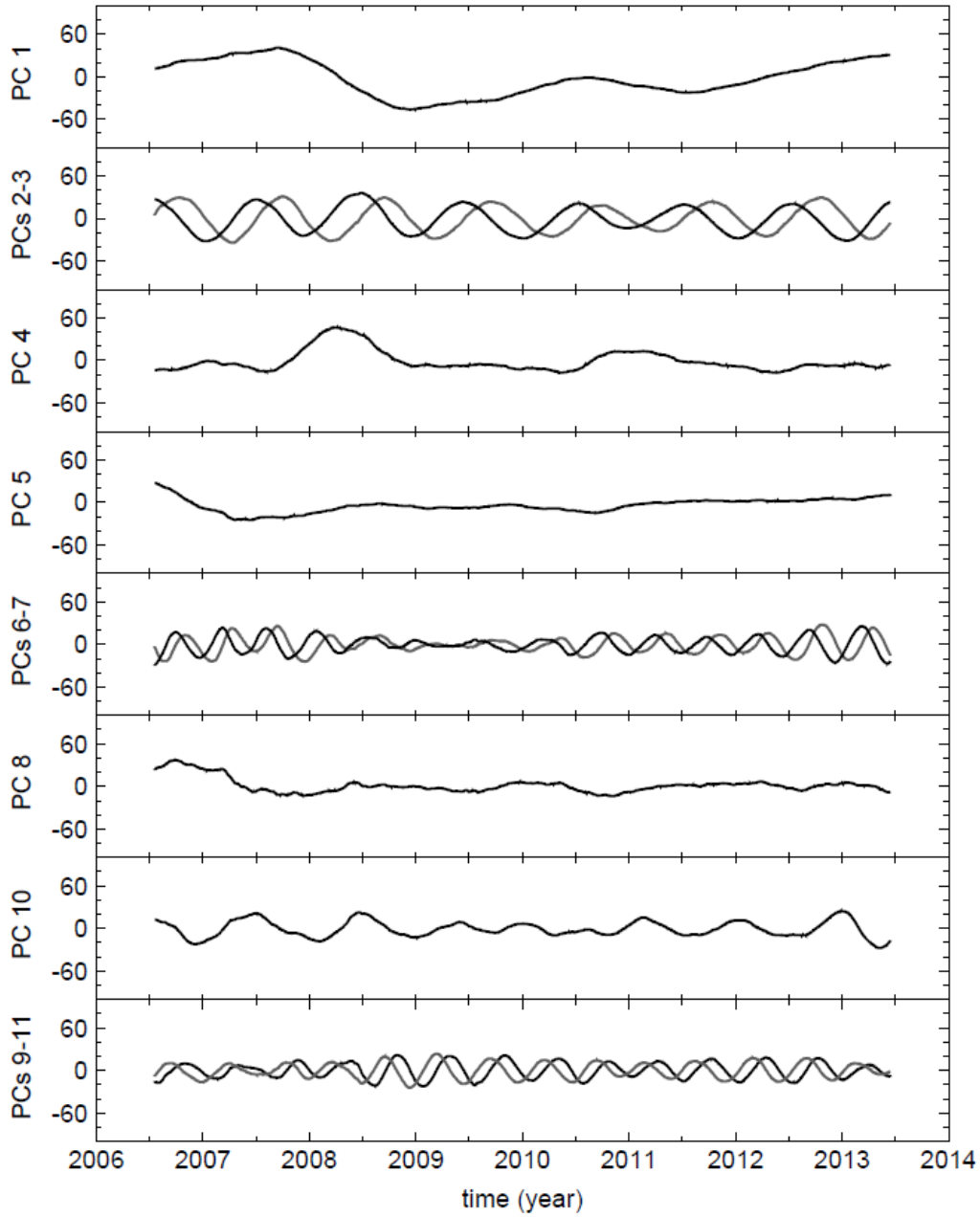
Akutan volcano



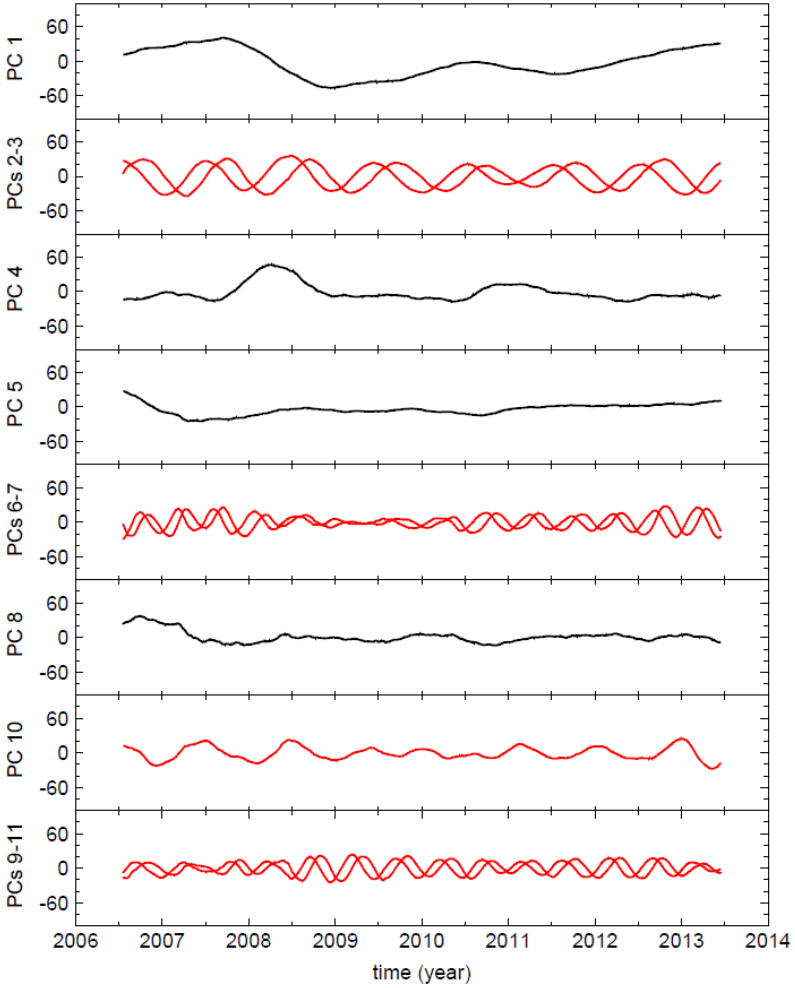
- Noise in the GPS time series
- Seasonal signal
- Transient signal sometimes readily visible

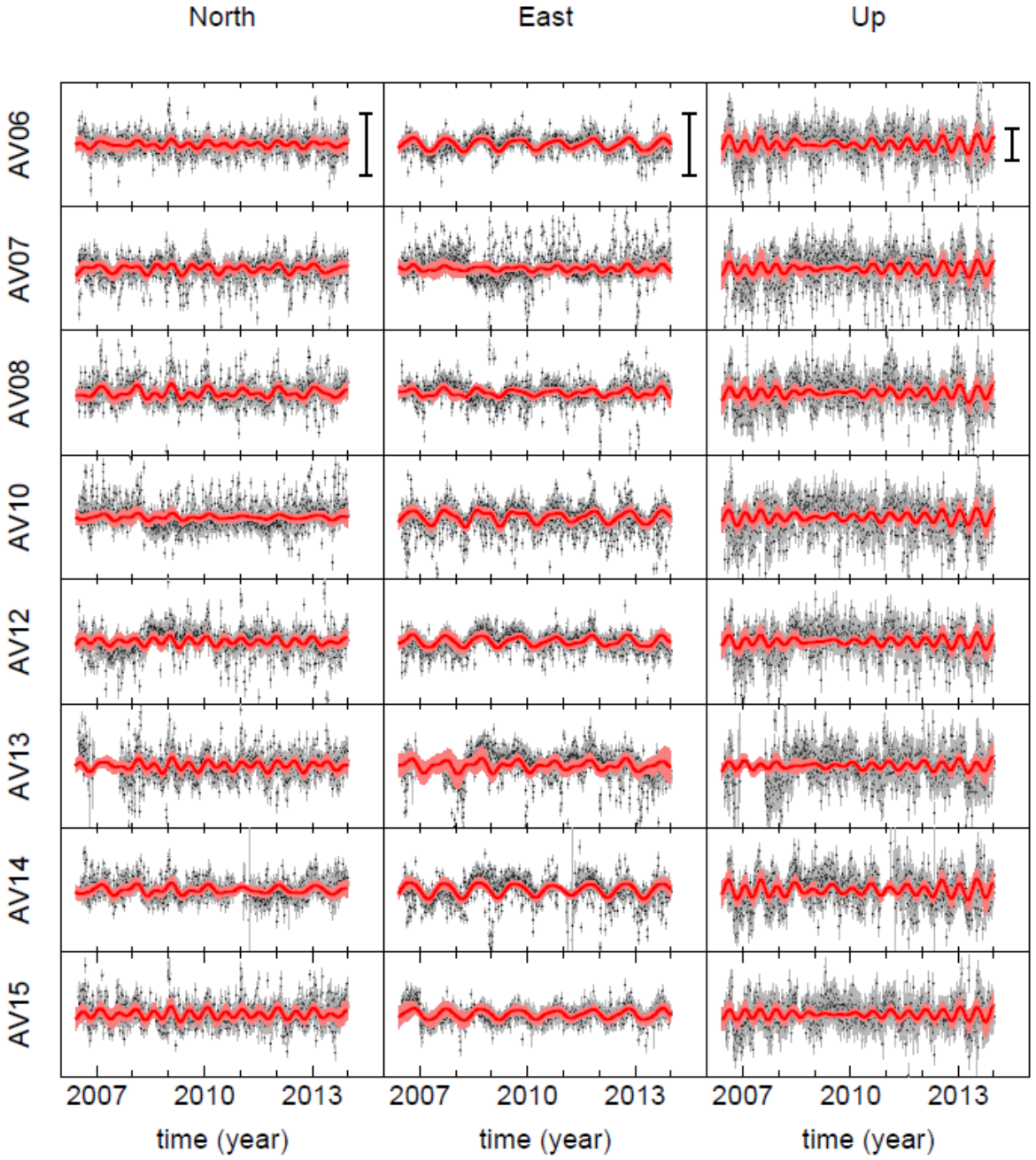
Akutan volcano



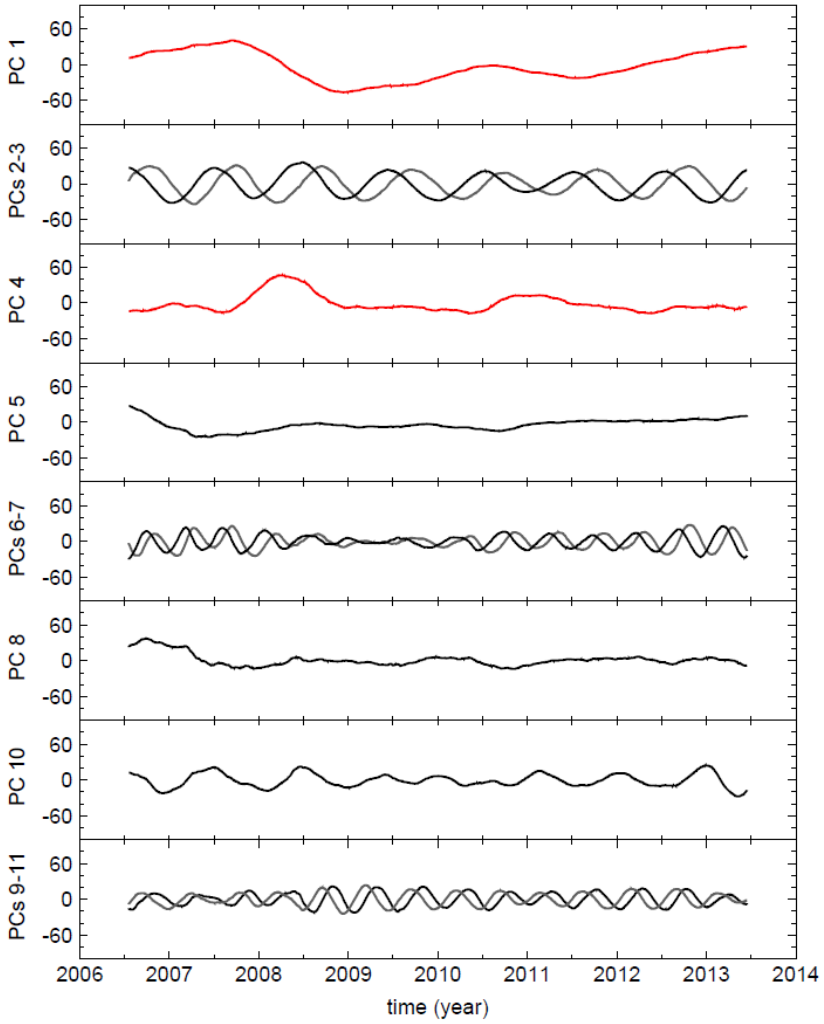


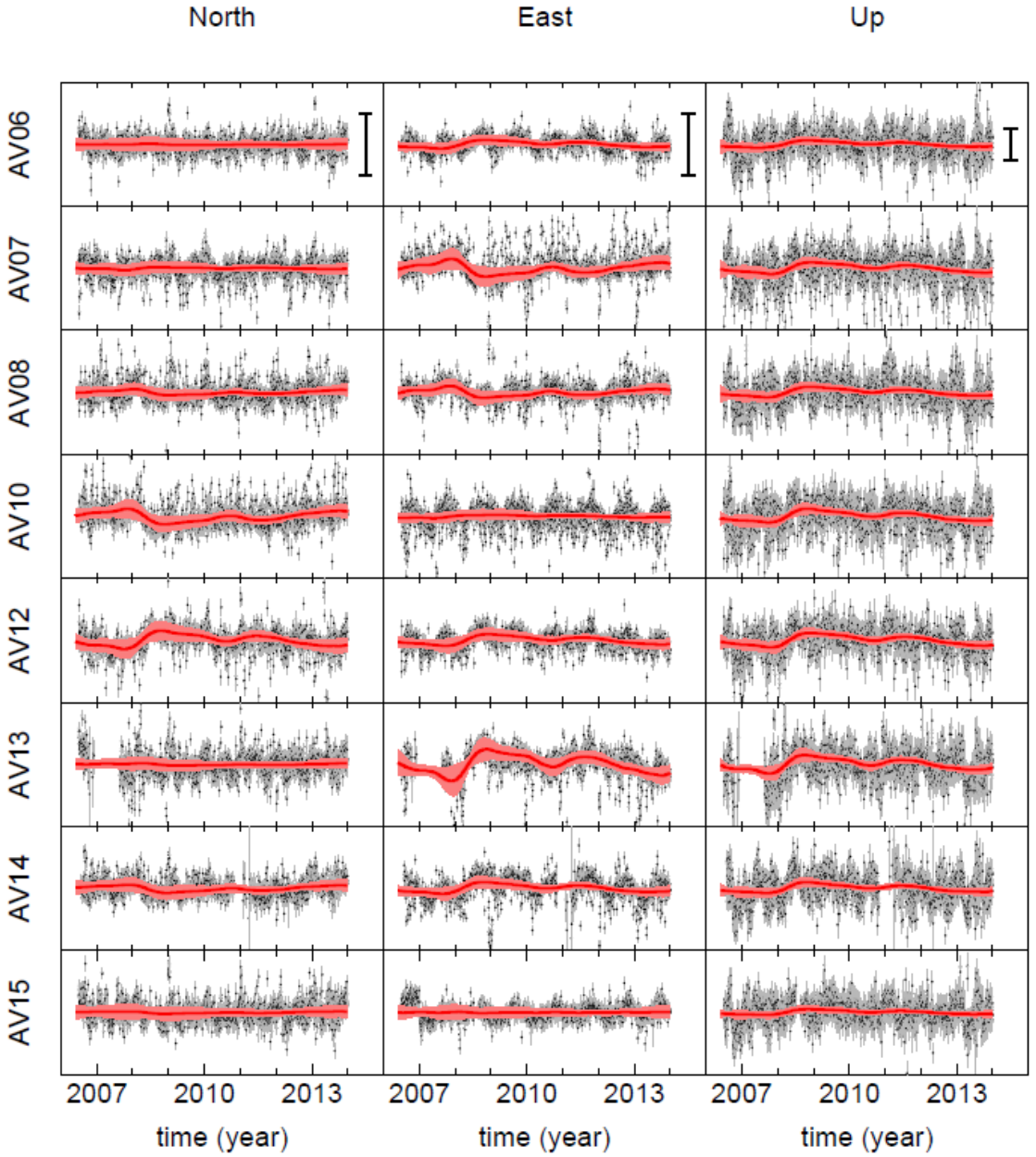
Akutan volcano



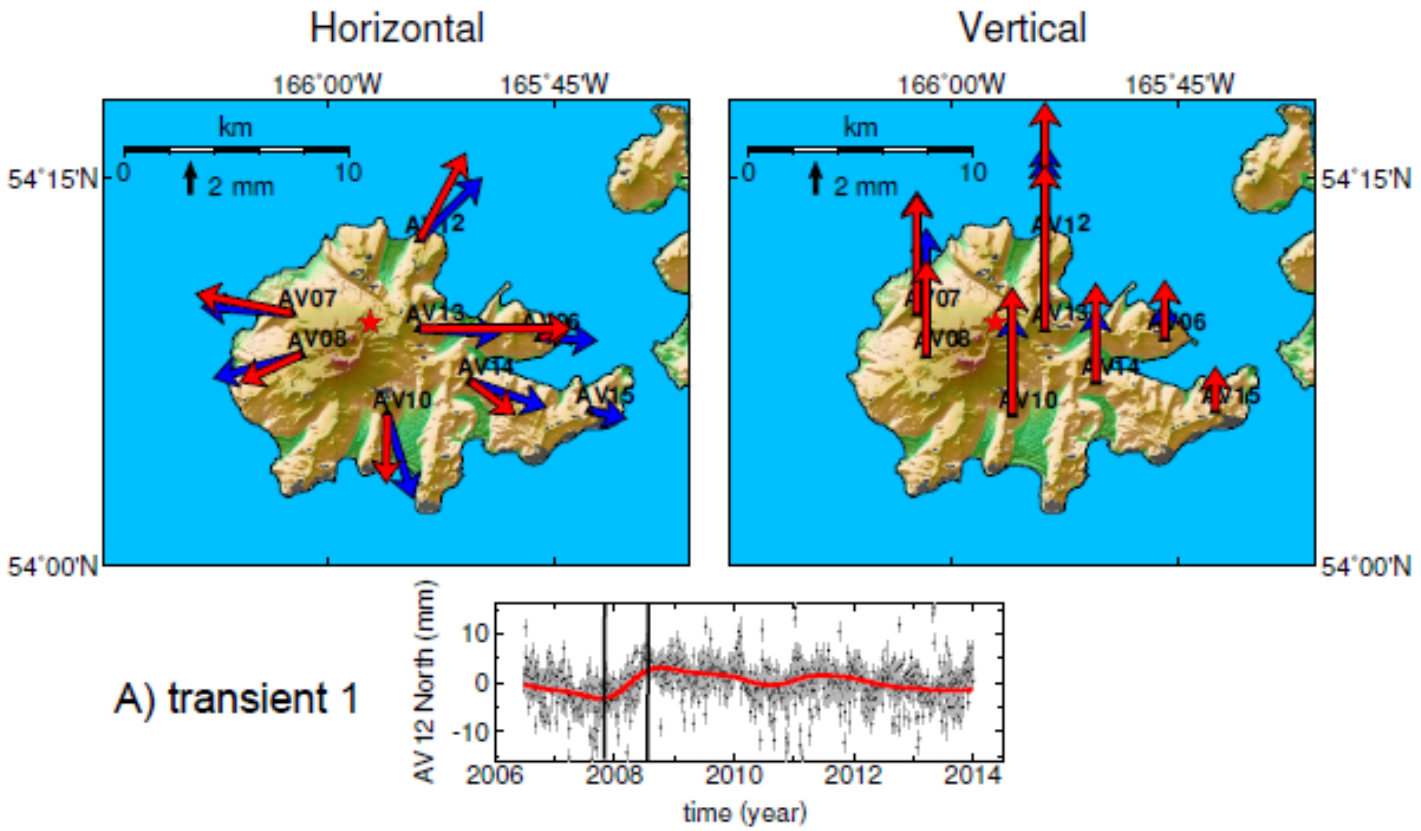


Akutan volcano

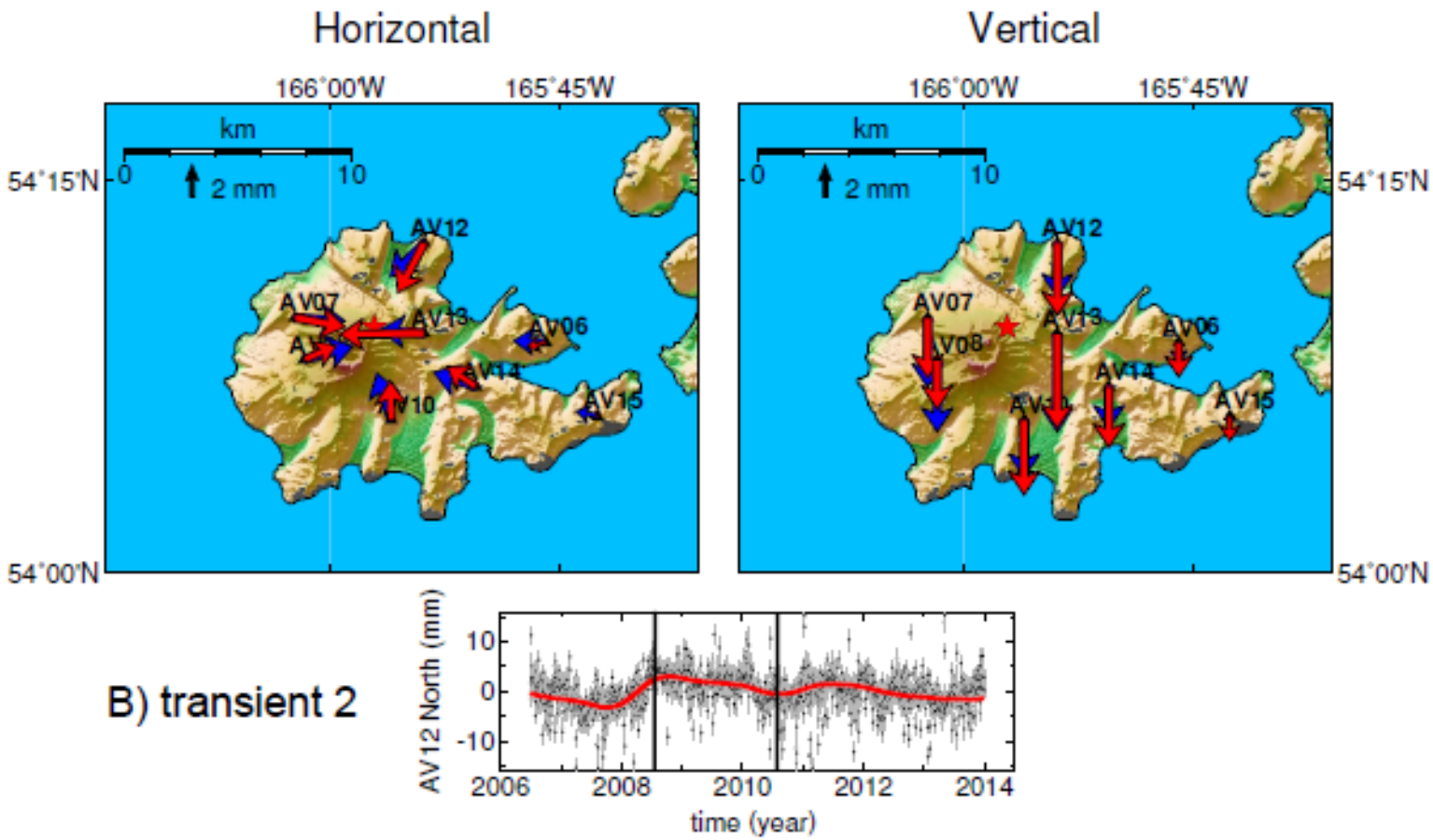




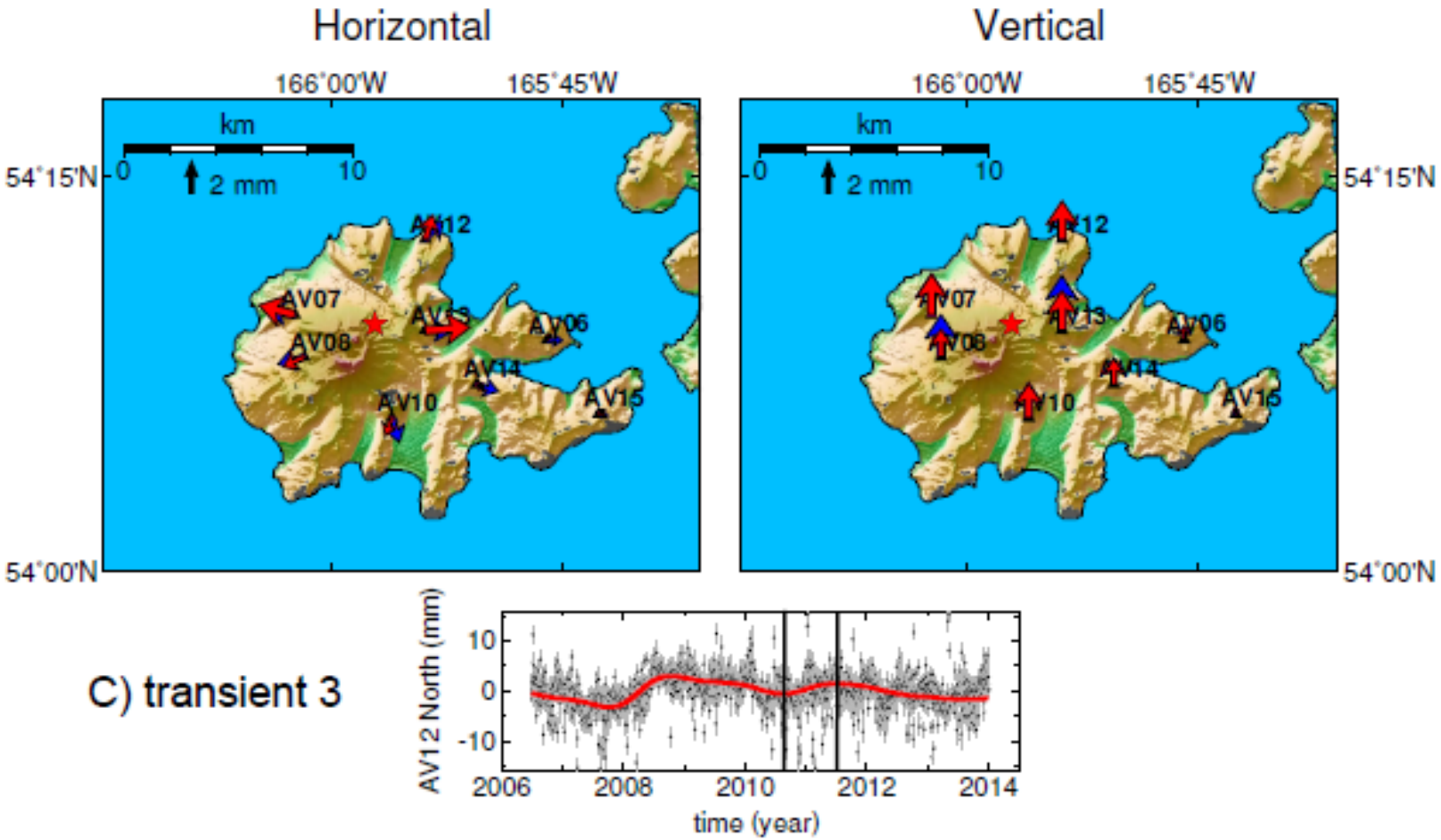
Akutan volcano



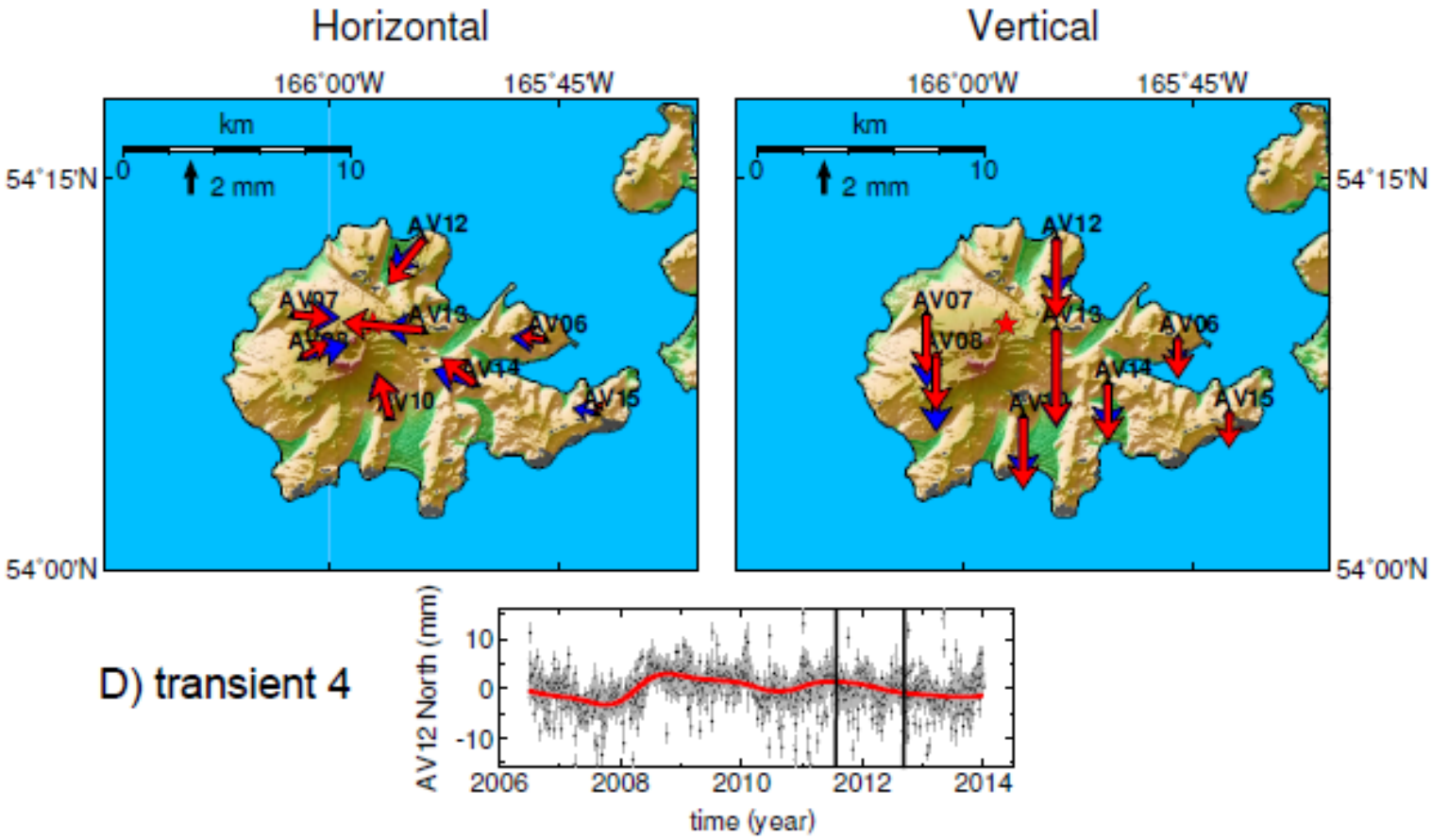
Akutan volcano



Akutan volcano



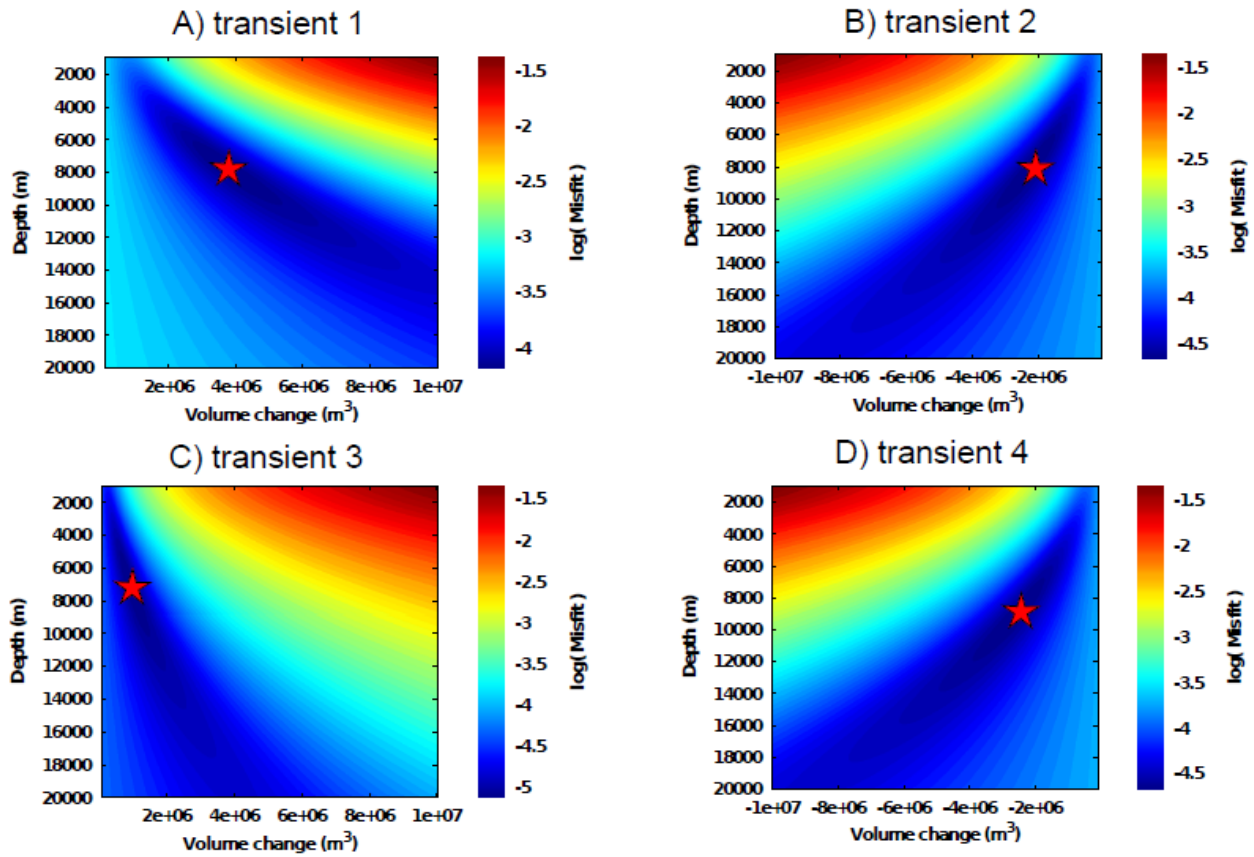
Akutan volcano

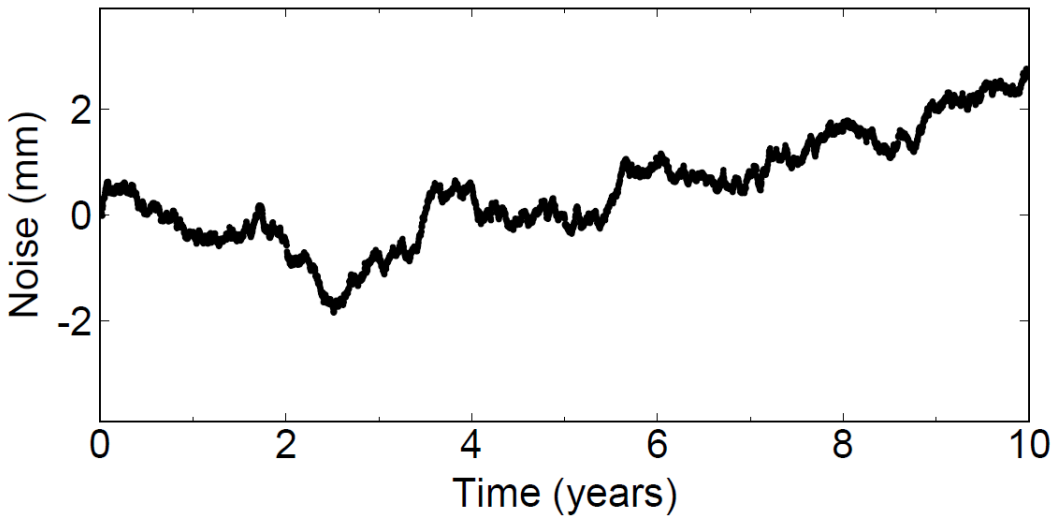


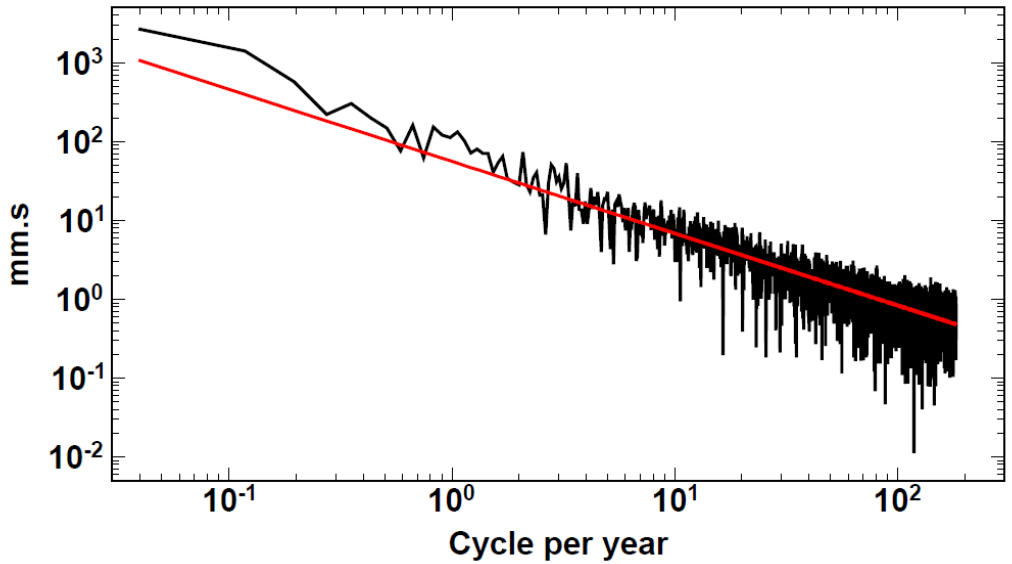
Conclusion

- M-SSA exploits spatial and temporal correlations of geophysical fields
- M-SSA is able to discriminate between different kind of signals
- M-SSA allows to extract from noise small transients displacements embeded in time series
- No use of a priori information

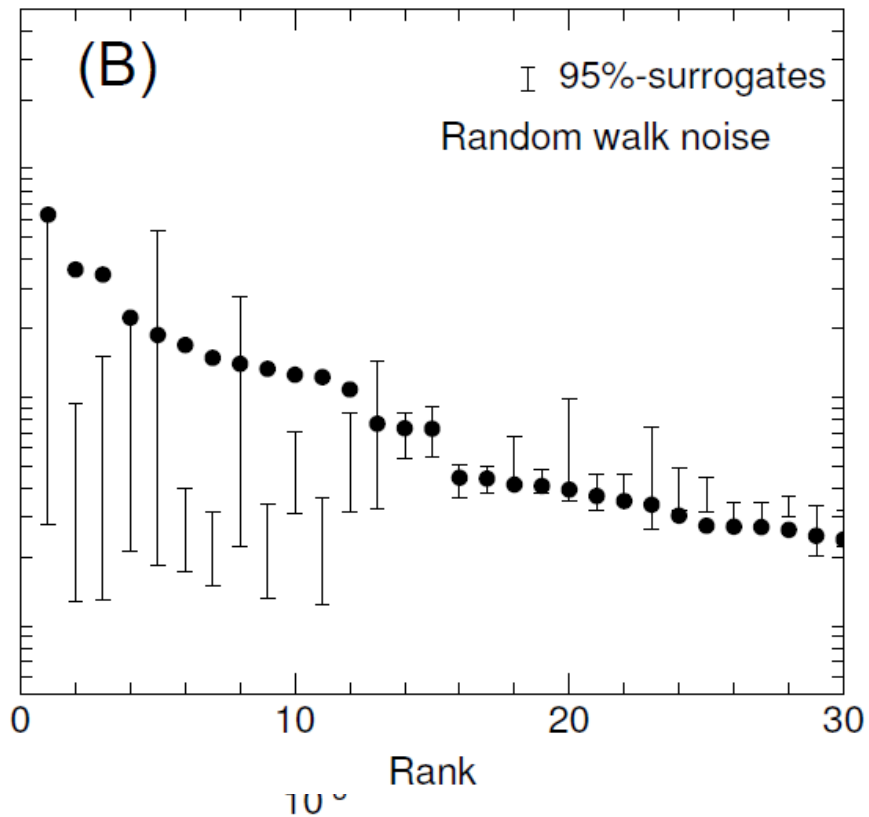
Reserved slides

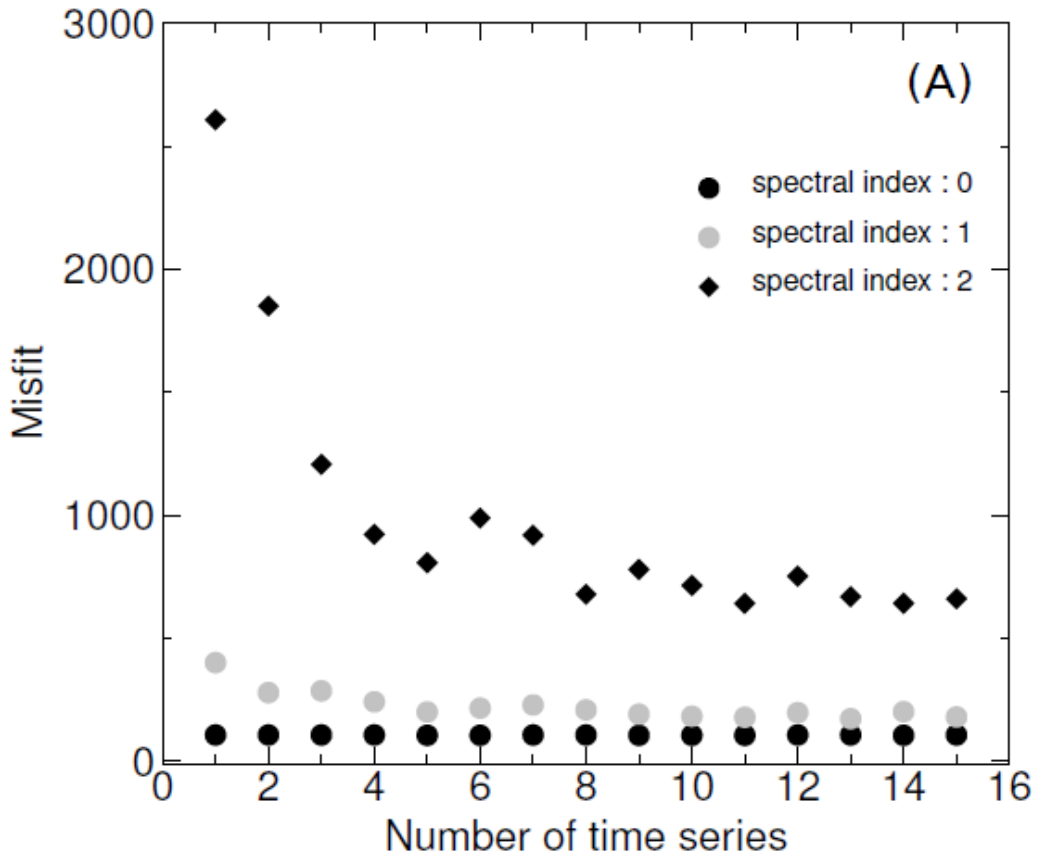

A) transient 1
B) transient 2
C) transient 3
D) transient 4
Depth (m)
Volume change (m3)
log(Misfit)
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
2e+06
4e+06
6e+06
8e+06
1e+07
-1e+07
-8e+06
-6e+06
-4e+06
-2e+06
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5

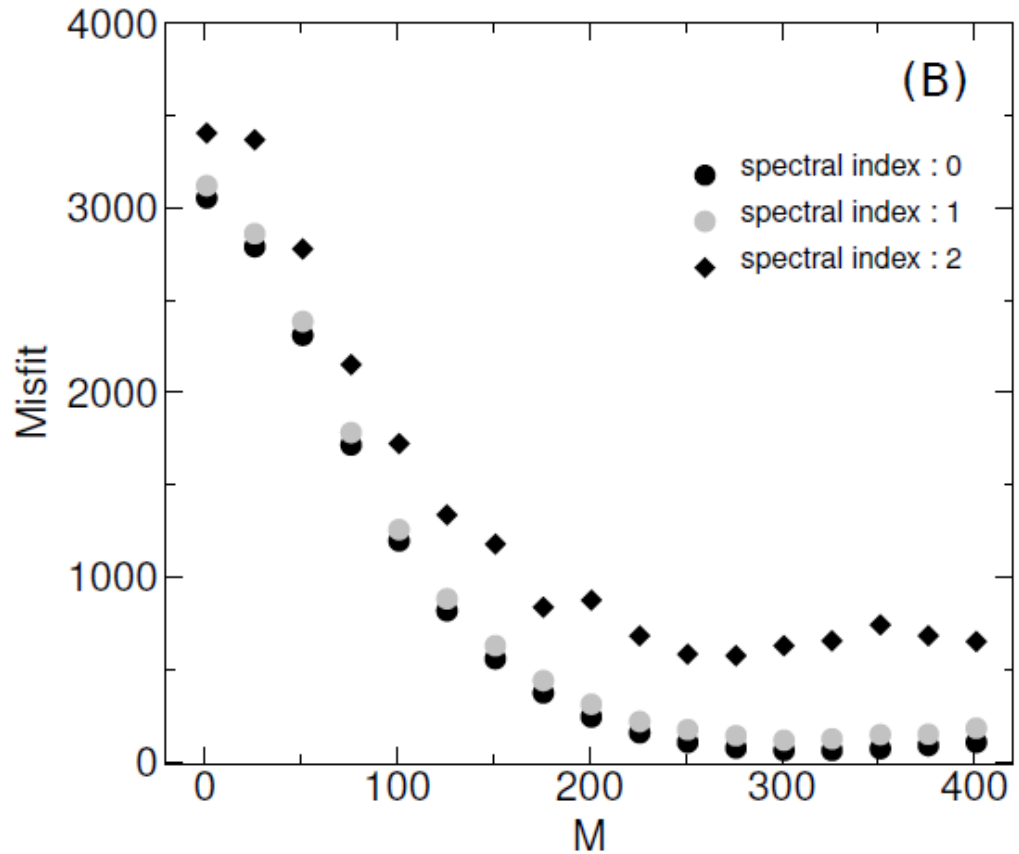


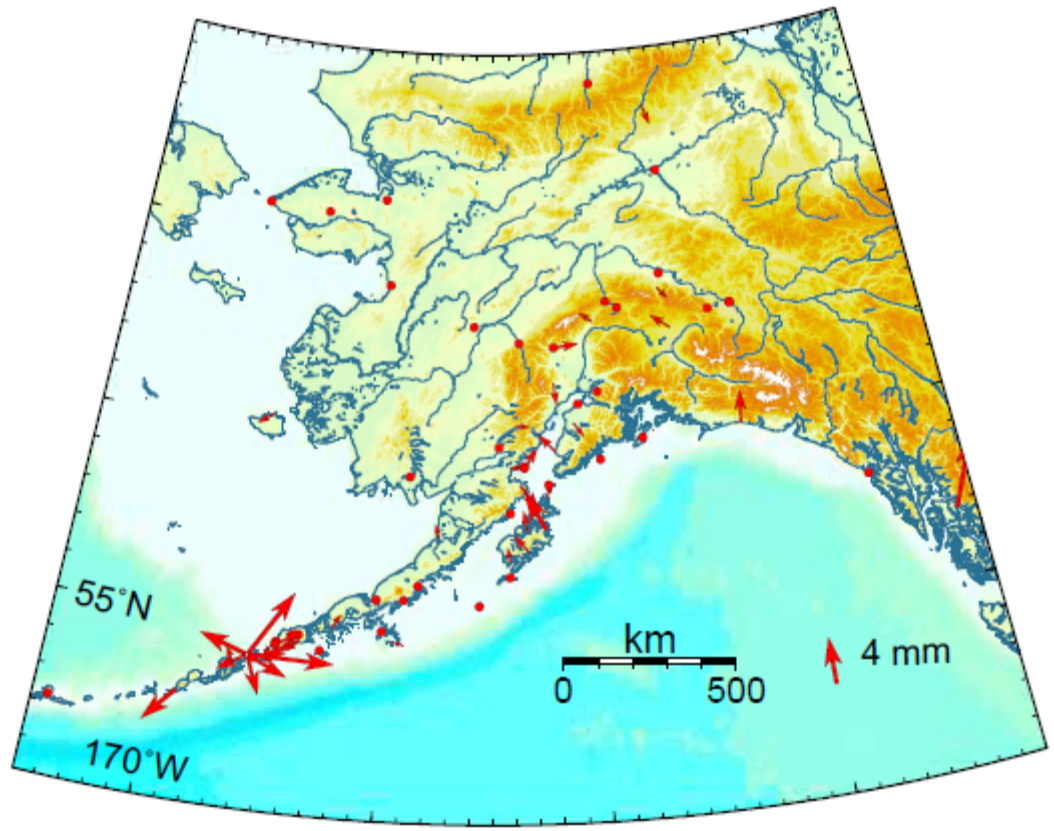


$$S(f) = S_0 \left(\frac{f}{f_0} \right)^{-\alpha}$$


(B)
95%-surrogates
Random walk noise
0
10
20
30
Rank

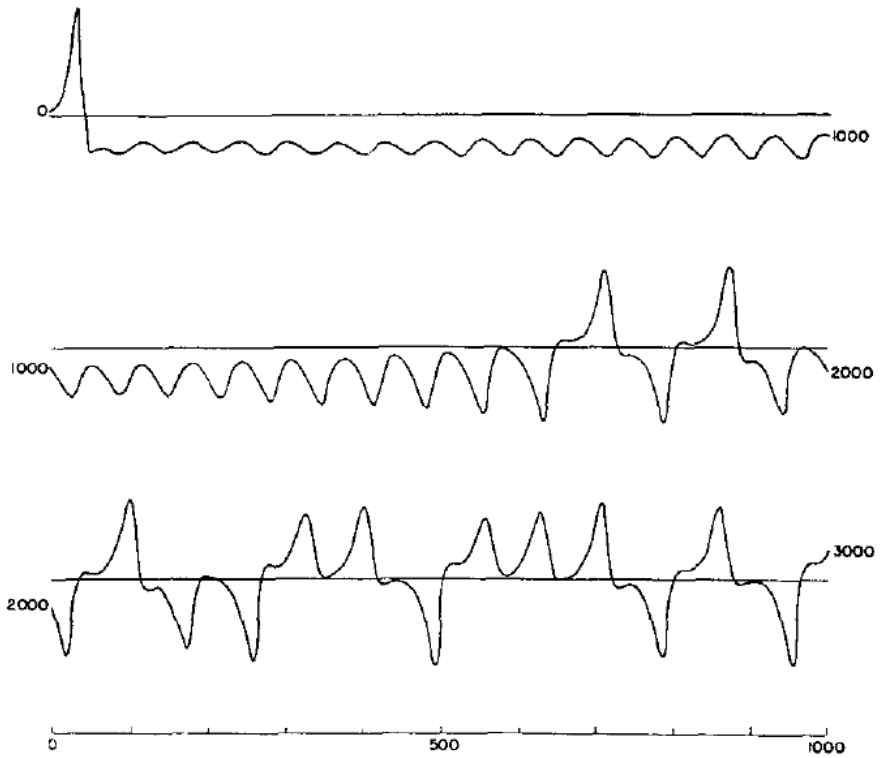

(A)
spectral index : 0
spectral index : 1
spectral index : 2
Misfit
Number of time series
0
1000
2000
3000
0
2
4
6
8
10
12
14
16

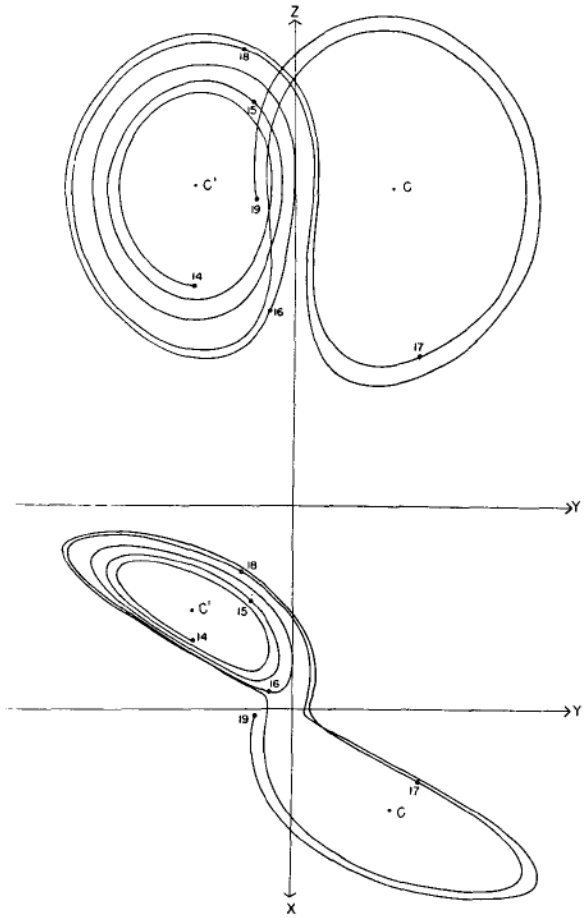

(B)
spectral index : 0
spectral index : 1
spectral index : 2
Misfit
M
0
1000
2000
3000
4000
0
100
200
300
400


55°N
170°W
km
0
500
4 mm

Time series and qualitative d

Transient deformation allow also to study the qualitative dynamics of geophysical systems





Saltzmann 1962, Lorenz 1963