Inversion of Geodetic data using least-square method in a non-Gaussian framework

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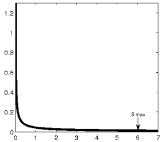
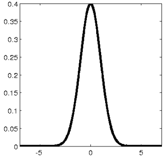
How to keep a Gaussian framework while using non Gaussian parameters ?

- to take outlier data into account
- to impose positivity or limit value

By defining a change of variable in the model or data space in order to obtain Gaussian new variables while the initial physical parameters present the desired non-Gaussian distribution.





Change of variable

Let s a parameter with pdf ρ. We define the Gaussien centered variable v by:

$$\rho(s)ds = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right) dv$$

It yields by integrating:

$$\int_{s_{med}}^{s} \rho(t)dt = \frac{1}{2}\mathrm{Erf}\left(\frac{v}{\sqrt{2}}\right)$$

$$\left(\text{Where} \quad \mathrm{Erf}(v) = \frac{2}{\sqrt{\pi}} \int_{0}^{v} \exp(-t^2)dt\right)$$

$$\text{And} \quad v(s) = \sqrt{2}\,\mathrm{Erf}^{-1}\left\{2\int_{s_{med}}^{s} \rho(t)dt\right\}$$

Inverting interseismic deformation in subduction areas

Using standard approach

- Back-slip (*Savage,1983*)
- Homogeneous elastic half-space (*Okada,1985*)
- Assuming that the subduction interface is perfectly known
- Taking account of information on converging direction

The problem is:

to invert ground displacement data for infering the field of slip over the subduction interface.

Inverting interseismic deformation in subduction areas (2)

$$d_i = \int_\Sigma k_i(x) \cdot \mathbf{s}(x) \mathrm{d}\Sigma(x) \quad \text{with} \quad s(x) \cdot n(x) = 0$$

with:

- Σ: the subduction interface
- $n(x)$: the normal unit vector to Σ at point x
- d_i: mesured displacement at a given station in a given direction
- $k_i(x)$: the corresponding kernel
- $\mathbf{s}$: the slip vector with horizontal azimuth θ and modulus s:

$$\mathbf{s} = \frac{s}{\sqrt{1+q^2}} \begin{pmatrix} \cos(\theta) \\ sin(\theta) \\ -q \end{pmatrix} \text{ where } q = \frac{n_x}{n_z} \cos(\theta) + \frac{n_y}{n_z} sin(\theta)$$

Bayesian approach

- The data $d = (d_i)_{i=1..n}$ are assumed to be Gaussian
- $\theta(s)$, is a Gaussian random function with mean, θ_{prior}, and covariance kernel: $Cov(x, x') = \sigma_x \sigma_{x'} \exp(\frac{||x-x'||}{\xi})$
- To impose positivity and to concentrate the field of displacement we assume that $s(x)$ follows a power law:

$$\rho(x) = p\frac{s^{p-1}}{s_{max}^{p}}\chi_{\{0,s_{max}\}}(s) \qquad s_{med} = s_{max}\frac{1}{2^p} \quad s_m = \frac{p}{p+1}s_{max} \qquad (0 < p \leq 1)$$

Consequently, we introduce the new field v such that:

$$\frac{1}{2}\text{Erf}\left(\frac{v}{\sqrt{2}}\right) = \int_{s_{med}}^{s} \rho(t)\text{d}t = \left(\frac{s}{s_{max}}\right)^p - \frac{1}{2} \qquad s = s_{max}\left(\frac{1}{2} + \frac{1}{2}\text{Erf}\left(\frac{v}{\sqrt{2}}\right)\right)^{1/p}$$

where $v(x)$ is assumed to be a centered Gaussian random function with covariance kernel: $\exp(\frac{||x-x'||}{\xi})$.

Non-linear standard least-squares algorithm

To minimize $||C_d^{-1/2}(d^{obs} - g(m))||^2 + ||C_m^{-1/2}(m - m_{prior})||^2$

$$m_{k+1} = m_{prior} + \delta_k \qquad \text{with}$$

$$\delta_k = C_m G_k^* (C_d + G_k C_m G_k^*)^{-1} (d_{obs} - g(m_k) + G_k(m_k - m_{prior}))$$

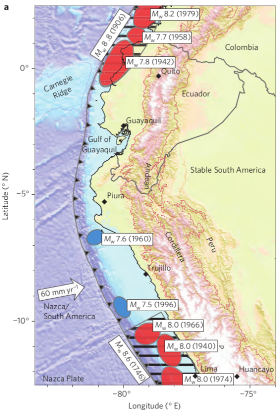
where

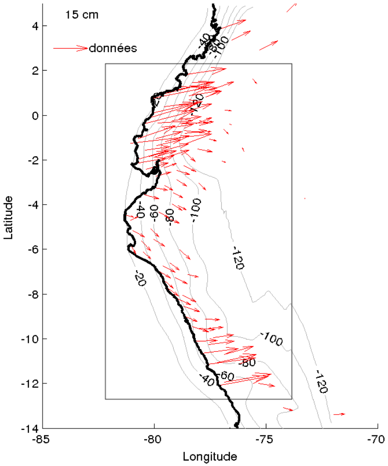
- $d_{obs} = (d_i)_{i=1..n}$: the vector of observed data
- C_d: covariance matrix representing error of mesurement (data space)
- G_k: the derivative operator of the function g at model m_k
- G_k^*: the adjoint of G_k
- $m = (\theta, v) \qquad m_{prior} = (\theta_{prior}, 0)$
- C_m: the covariance operator (model) with kernel $\begin{pmatrix} \sigma_\theta & 0 \\ 0 & 1 \end{pmatrix} \exp(\frac{||x-x'||}{\xi})$

Actually, we use:

$$m_{k+1} = \alpha_k m_k + (1 - \alpha_k)(m_{prior} + \delta_k) \quad \text{with} \quad 0 \leq \alpha_k < 1$$

Northern Andean subduction zone





- Interseismic data (from *Nocquet et al. 2014*)
- 100 horizontal GPS vectors

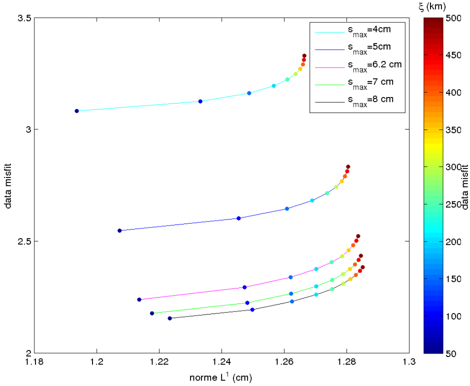
Choice of parameters

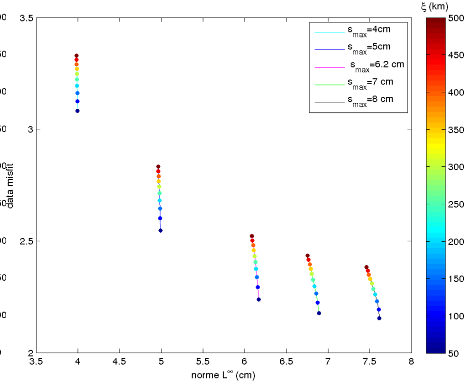
Tuning parameters

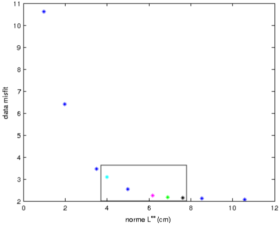
- Maximum slip: $s_{max} \in [1cm, 20cm]$
- Power parameter: $p \in]0, 1]$
- Prior correlation lenght: $\xi \in [50km, 500km]$

- Azimuth: $\theta_{prior} = N83^\circ \quad \sigma_\theta = 1^\circ$
- Interface of subduction: geometry from slab 1.0 model (*Hayes, G. P., Wald, D. J., Johnson, R. L., 2012*)

varying ξ for various maximum slip with $p = 1/3$

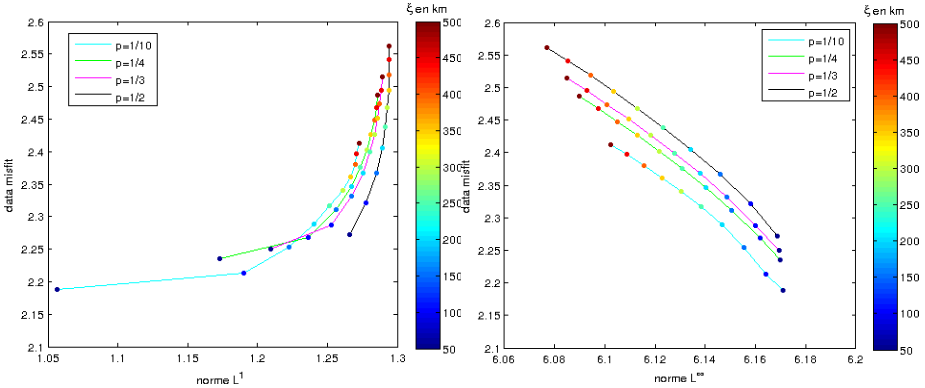






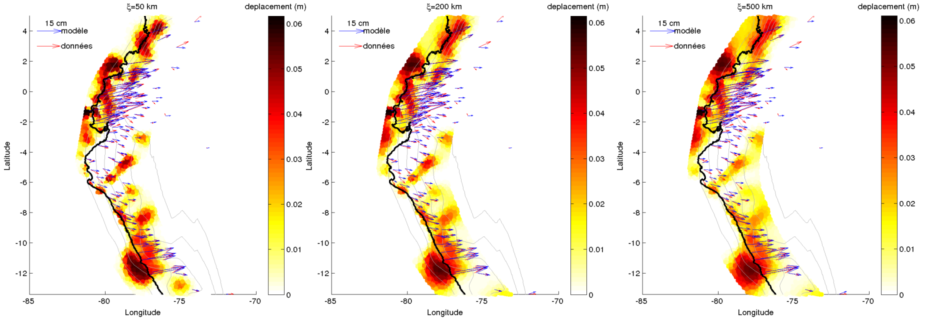
- The data fitting is mostly controlled by s_{max}

varying ξ for various p with $s_{max} = 6.2cm$



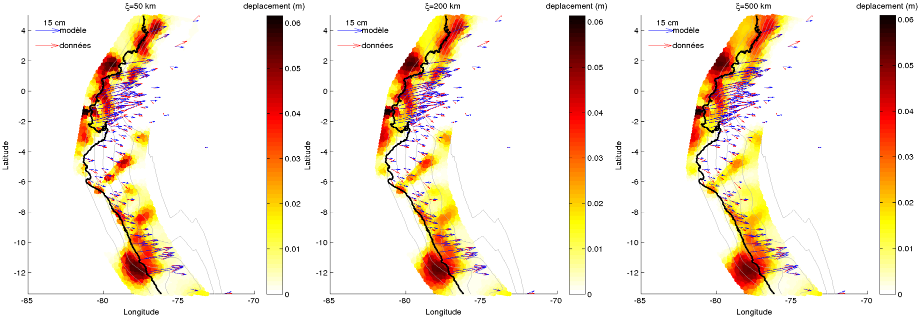
- The smaller p or ξ is, the better the data fitting
- The maximum displacement increases when p or ξ decreases

Models for $p = 1/10$ and $s_{max} = 6.2cm$ with various ξ

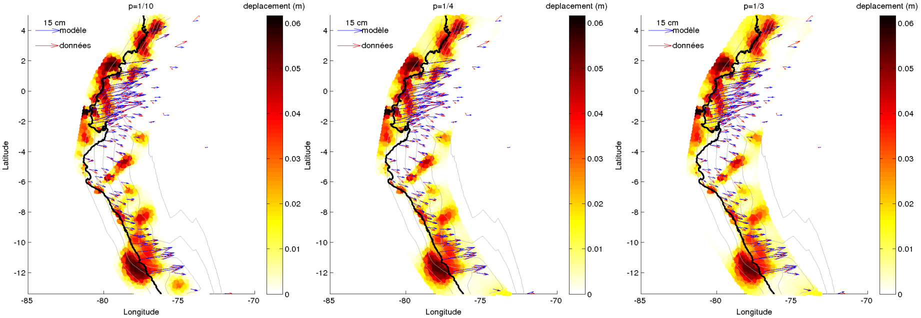


- The slip is more concentrated for small smoothing

Models for $p = 1/4$ and $s_{max} = 6.2cm$ with various ξ

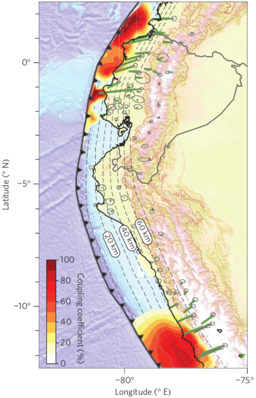


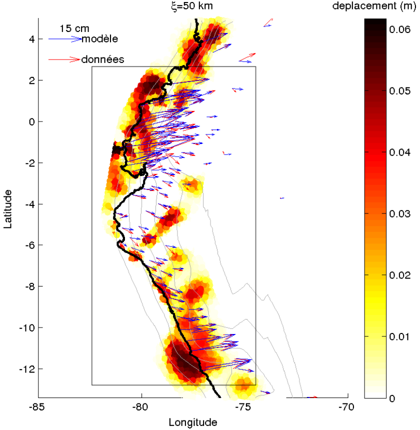
Models for $\xi = 50km$ and $s_{max} = 6.2cm$ with various p



- The concentration of the slip is better for small p

Comparison with *Nocquet et al. 2014*





- We have inverted directly the GPS data without previously removing a two-block solid rotation.
- Same kind of results in Northern Ecuador and in Central and Southern Peru.

THANK YOU FOR YOUR ATTENTION